Formal Methods (8803 B, Spring 2004) Panagiotis Manolios
Notations handout 1/6/2004

These notes are concerned with notational issues. A warning. The notation I use differs from the notation you will see in the books we will use for the course. Being exposed to various notational conventions is a good thing, but may lead to confusion, so if something is not clear, please ask!

1 Initial Notation and Definitions

$\mathbb{N}$ and ω both denote the natural numbers, *i.e.*, $\{0, 1, \ldots\}$. The ordered pair whose first component is i and whose second component is j is denoted $\langle i, j \rangle$. $[i..j]$ denotes the closed interval $\{k \in \mathbb{N} \ : \ i \leq k \leq j\}$; parentheses are used to denote open and half-open intervals, *e.g.*, $[i..j)$ denotes the set $\{k \in \mathbb{N} \ : \ i \leq k < j\}$.

R is a *binary relation* on set S if $R \subseteq S \times S = \{\langle x, y \rangle : x, y \in S\}$. We abbreviate $\langle s, w \rangle \in R$ by sRw. A function is a relation such that xRy and xRw implies $y = w$.

Function application is sometimes denoted by an infix dot "." and is left associative. That is, $f.x$ is the unique y such that xfy. This allows us to use the curried version of a function when it suits us, *e.g.*, we may write $f.x.y$ instead of $f(x, y)$. That is, $f.x.y$ is really $(f.x).y$, where f is a function of one argument that returns $f.x$, a function of one argument.

From highest to lowest binding power, we have: parentheses, function application, binary relations (*e.g.*, sBw), equality ($=$) and membership ($\in$), conjunction ($\wedge$) and disjunction ($\vee$), implication ($\Rightarrow$), and finally, binary equivalence ($\equiv$). Spacing is used to reinforce binding: more space indicates lower binding.

$\langle Qx : r : b \rangle$ denotes a quantified expression, where Q is the quantifier, x the bound variable, r the range of x (**true** if omitted), and b the body. We sometimes write $\langle Qx \in X : r : b \rangle$ as an abbreviation for $\langle Qx : x \in X \ \wedge \ r : b \rangle$, where r is **true** if omitted, as before.

Cardinality of a set S is denoted by $|S|$. $\mathcal{P}(S)$ denotes the powerset of S.

A function from $[0..n)$, where n is a natural number, is called a *finite sequence* or an *n-sequence*.

What are numbers as mathematical objects? von Neumann proposed the following: $0 = \emptyset, 1 = \{0\}, 2 = \{0, 1\}, \ldots$, so $n = [0..n)$. Thus an n-sequence is a function from n.

An *ω-sequence* is a function from ω. We may sometimes refer to ω-sequences as infinite sequences, but as we will see there are infinite sequences that are "longer" than ω-sequences.

When we write $x \in \sigma$, for a sequence σ, we mean that x is in the range of σ.