EM algorithm for coin flipping problem

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We have K coins. The mixing proportions are $\pi_1, \pi_2, \ldots, \pi_K$. The probability of the k's coin getting heads is q_k. First we randomly pick a coin, then we flip this coin for d times. And we repeat this process for N times. In this way, we can generate N data points $\{x_1, x_2, \ldots, x_N\}$, each of which is a d dimensional vector. $x_i \in \{Head, Tail\}^d$. Suppose the number of heads in x_k is y_k. We define the hidden variables z_n, representing the component assignment for data point x_n using a vector of size K. If x_n is drawn from the kth component, $z_{nk} = 1$ while the remaining are all 0.

- E step: Compute $\gamma(z_{nk})$ with current parameters $\theta = \{\pi_k, q_k\}$.

$$\gamma(z_{nk}) = p(z_{nk} = 1 | x_n, \theta) = \frac{\pi_k p(x_n | q_k)}{\sum_{j=1}^{K} \pi_j p(x_n | q_j)}$$

- M step: update π_k and q_k

$$\pi_k = \frac{\sum_{n=1}^{N} \gamma(z_{nk})}{N}$$

$$q_k = \frac{\sum_{n=1}^{N} \gamma(z_{nk}) y_n}{\sum_{n=1}^{N} \gamma(z_{nk}) d}$$

References

[1] Dawen Liang, Technical Details about the Expectation Maximization (EM) Algorithm, 2012