CS1800
Discrete Structures
Fall 2017

Lecture 6
9/18/17

Last time

- truth tables
- logic gates
- circuit construction
  - DNF
- logical equivalence
  - laws of Boolean algebra

Today

- arithmetic circuits
  - half-adder
  - full-adder
  - ripple-carry adder

- logic
  - handout

Next time

- Module 2:
  Encryption

a	b	out	out'
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	0

DNF construction

$$out = (\neg a \wedge \neg b) \vee (\neg a \wedge b) \vee (a \wedge b)$$

$$\vdots \qquad \text{(laws of Boolean algebra)}$$

$$= \neg a \vee b$$

$$out' = a \wedge \neg b$$

$$out = \neg out' = \neg (a \wedge \neg b)$$

$$= \neg a \vee \neg\neg b$$

$$= \neg a \vee b \quad \checkmark$$

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

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a	b	out	out'
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

DNF: $out = (\neg a \wedge b) \vee (a \wedge \neg b)$

$$out' = (\neg a \wedge \neg b) \vee (a \wedge b)$$

$$out = \neg out' = \neg [(\neg a \wedge \neg b) \vee (a \wedge b)]$$

$$= \neg(\neg a \wedge \neg b) \wedge \neg(a \wedge b)$$

$$= (a \vee b) \wedge (\neg a \vee \neg b)$$

CNF

Half-adder

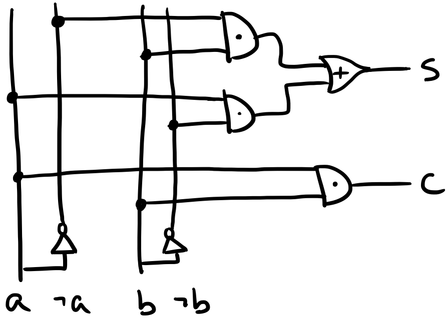
	inputs		outputs
a	b	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

1 carry

a 1 0 1 ← inputs

b + 0 1 1

0 sum



inputs

outputs

DNF construction:

$$s = (\neg a \wedge b) \vee (a \wedge \neg b)$$

$$c = a \wedge b$$

Full-adder

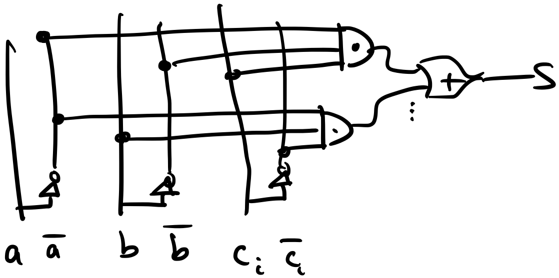
carry out

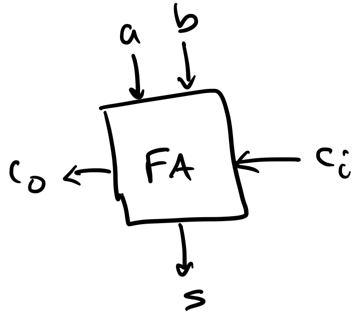
$$\begin{array}{rccc} & 1 & 1 & 0 \\ & 1 & 1 & 0 \\ + & 1 & 1 & 1 \\ \hline & 1 & 0 & 1 \end{array}$$

← 3 inputs

sum

a	b	c_i	s	c_o
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
⋮	⋮	⋮	⋮	
1	1	1	1	1



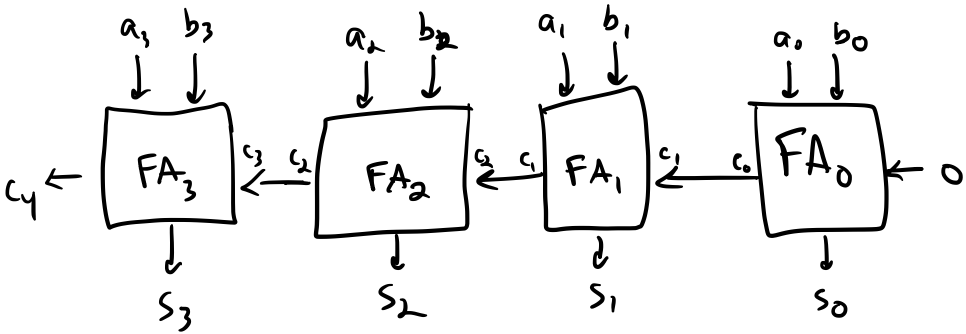


$$s = (\neg a \wedge \neg b \wedge c_i) \vee (\neg a \wedge b \wedge \neg c_i) \cdots$$

$$c_o = (\neg a \wedge b \wedge c) \vee \cdots$$

Ripple-carry adder

$$\begin{array}{r} 1011 \\ +\ 1100 \\ \hline \end{array}$$



Formal Logic

if-then $\longrightarrow$ implications
if and only if

① Implies: $A \Rightarrow B$ "A implies B"

A : "hot outside"

B : "not many people outside"

implication: $A \Rightarrow B$ "if hot outside, then not many people outside"

converse: $B \Rightarrow A$ converse is not <u>necessarily</u> true.

contrapositive: $\neg B \Rightarrow \neg A$ "if many people outside, then not hot outside"

* implication and it's contrapositive are logically equivalent

- Can chain implications together

$A \Rightarrow B$, $B \Rightarrow C$ $\Rightarrow$ $A \Rightarrow C$ ← at least 45 min to get home.

A: Friday afternoon; B: traffic is bad; B: traffic bad; C: at least 45 min to get home; A: fri afternoon

$$(A \Rightarrow B) \wedge (B \Rightarrow C) \Rightarrow (A \Rightarrow C)$$

- $(P \wedge Q) \Rightarrow (R \vee S \vee \neg T)$

- $P \Rightarrow Q \quad \equiv \quad \neg P \vee Q$

$$P \wedge Q \wedge (P \wedge Q \Rightarrow R) \quad \therefore \quad P \wedge Q \wedge R$$

$$\begin{aligned}
P \wedge Q \wedge (P \wedge Q \Rightarrow R) &\equiv P \wedge Q \wedge (\neg(P \wedge Q) \vee R) \\
&\equiv P \wedge Q \wedge (\neg P \vee \neg Q \vee R) \\
&\equiv P \wedge (Q \wedge \neg P \vee Q \wedge \neg Q \vee Q \wedge R) \equiv P \wedge (Q \wedge \neg P \vee F \vee Q \wedge R) \\
&\equiv P \wedge (Q \wedge \neg P \vee Q \wedge R)
\end{aligned}$$

$$
\begin{aligned}
P \wedge (Q \wedge \neg P \vee Q \wedge R) &= P \wedge Q \wedge \neg P \vee P \wedge Q \wedge R \\
&= F \vee P \wedge Q \wedge R \\
&= P \wedge Q \wedge R
\end{aligned}
$$