Last time

- Finished two's comp.
- Start circuits & logic

Today

- Truth tables
- Logic gates
- Circuits
  - construction
  - DNF, CNF
- Logical Equivalence
  - Laws of Boolean algebra
- Logical Completeness
- Circuits for addition
  - half-adder
  - full-adder
  - ripple-carry adder

Next time

- Propositional & Predicate Logic

Motivating example: binary addition

carry

$$
\begin{array}{lrrrr}
a: & & 1 & 0 & 1 \\
b: & + & 1 & 1 & 0 \\
\hline
 & 1 & 0 & 1 & 1
\end{array}
\qquad\qquad
\begin{array}{rrrrr}
 & & 1 & 1 & 1 \\
+ & & 1 & 0 & 1 \\
\hline
 & 1 & 1 & 0 & 0
\end{array}
$$

sum

a_0	b_0	s_0
0	0	0
0	1	1
1	0	1
1	1	0

a_0	b_0	c_0
0	0	0
0	1	0
1	0	0
1	1	1

truth tables

$s_0 = 1$ iff $(a_0=0 \text{ AND } b_0=1) \text{ OR } (a_0=1 \text{ AND } b_0=0)$

$(a_0=\text{NOT } 1 \text{ AND } b_0=1) \text{ OR } (a_0=1 \text{ AND } b_0=\text{NOT } 1)$

Boolean function

analogy

$f(x) = x^2$

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4

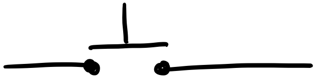
AND, OR, NOT

0	0V	F	no current open
1	+5V	T	current closed

a	b	a AND b
0	0	0
0	1	0
1	0	0
1	1	1

$a \wedge b$, $a \cdot b$, 

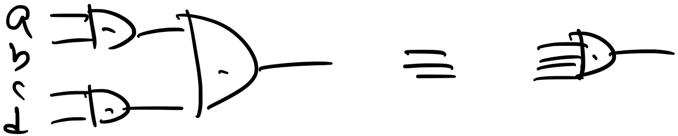
Digression: **Switches**



normally open
push to close switch

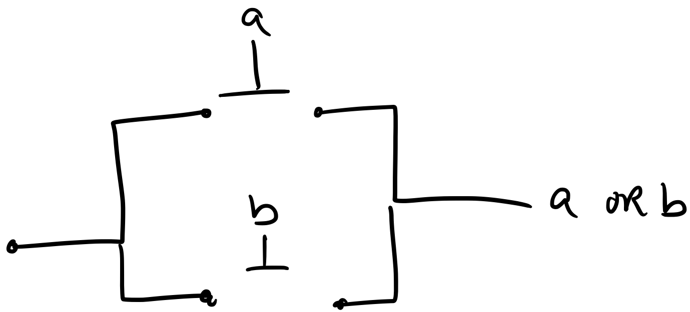
normally closed
push to open switch

a AND b

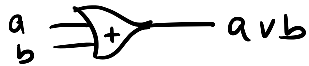


OR

a	b	a OR b
0	0	0
0	1	1
1	0	1
1	1	1

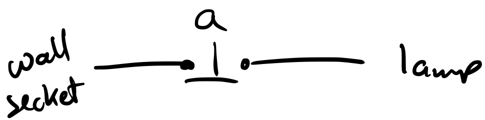


OR, +, $a \vee b$

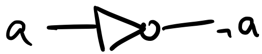


NOT

a	NOT A
0	1
1	0



NOT, $\neg a$, $\overline{a}$

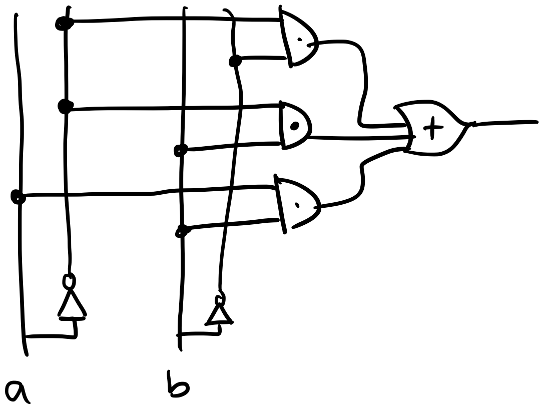


Laws of (Boolean) Logic/Algebra

	nomal algebra	Boolean algebra
comm. law	$a+b = b+a$	$a \vee b = b \vee a$
	$a \cdot b = b \cdot a$	$a \wedge b = b \wedge a$
distr.	$a \cdot (b+c) = a \cdot b + a \cdot c$	$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$
		$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
ident.	$a \cdot 1 = a$	$p \wedge T = p$
	$a + 0 = a$	$p \vee F = p$
De Morgan		$\neg(p \wedge q) = \neg p \vee \neg q$
		$\neg(p \vee q) = \neg p \wedge \neg q$

DNF construction

a	b	out
0	0	1
0	1	1
1	0	0
1	1	1



$$\equiv \quad (\neg a \wedge \neg b) \vee (\neg a \wedge b) \vee (a \wedge b)$$

$$\stackrel{?}{\equiv} \quad \neg a \vee b$$

a	b	$\neg a \vee b$
0	0	1
0	1	1
1	0	0
1	1	1

Example (Logical Equivalence)

$$(\neg a \wedge \neg b) \vee [(\neg a \wedge b) \vee (a \wedge b)]$$

$$\equiv (\neg a \wedge \neg b) \vee ((\neg a \vee a) \wedge b)$$

$$\equiv (\neg a \wedge \neg b) \vee (T \wedge b)$$

$$\equiv (\neg a \wedge \neg b) \vee b$$

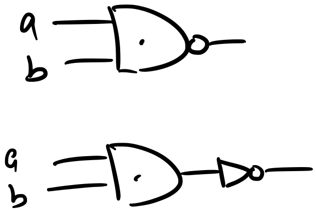
$$\equiv (\neg a \vee b) \wedge (\neg b \vee b)$$

$$\equiv (\neg a \vee b) \wedge T$$

$$\equiv \neg a \vee b$$

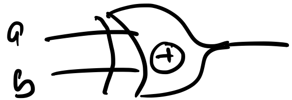
NAND

a	b	a NAND b
0	0	1
0	1	1
1	0	1
1	1	0



XOR

a	b	a XOR b
0	0	0
0	1	1
1	0	1
1	1	0



$\oplus$

$a \oplus b$