Last time

- Counting in binary
- Powers of two
- Hex / Oct
  - conversions
- Negative #'s
  - Signed mag.
  - One's comp.
  - two's comp.

Today

- Finish two's comp.
  - conversions
  - examples
  - intuition
- Circuits
  - switches
  - logic gates
  - truth tables

Next time

- Binary arithmetic
- Logical equivalence
- Normal forms

2's Complement : Conversion rules

From dec → 2's comp.

		4-bit 2's comp. (4 2 1)	math
① Pos # → rep. in binary	e.g 6	0 1 1 0	x
② Neg # → ⓐ mag. in binary	e.g -6	0 1 1 0	x
ⓑ flip all bits		1 0 0 1	$(2^n-1)-x$
ⓒ add 1		1 0 1 0	$(2^n-1)-x+1$
			$=2^n-x$

e.g $2^4-6=10$

From 2's comp → dec

By convention, ~~first~~ MSB bit is a sign bit

0 → +

1 → −

① leading 0 → pos # → bits are mag. in binary

e.g. 0101 → +5

② leading 1 → neg # → convert bits back to determine mag.

one way

$$\begin{array}{r} 1\ 1\ {}^{0}\!\not{1}\ 0 \\ -\ 0\ 0\ 0\ 1 \\ \hline 1\ 1\ 0\ 1 \end{array}$$

subtract 1

$$0\ 0\ 1\ 0$$

flip bits

$\equiv 2_{10} \Rightarrow -2$

$1\ 1\ 1\ 0$

$0\ 0\ 0\ 1$ flip bits $(2^n-1)-(2^n-x)$

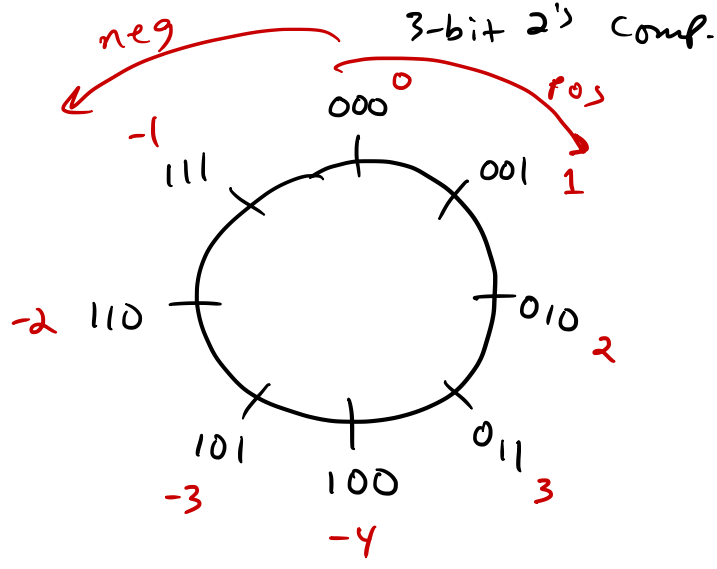
$0\ 0\ 1\ 0$ add 1 $(2^n-1)-(2^n-x)+1$

$= x$

$2^n - x$

$2^4 - 2 = 16 - 2 = 14$

#	
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111



largest pos: $2^{n-1} - 1$

most neg: -2^{n-1}

Add & Sub : 4-bit 2's comp.

3: 0011

4: 0100

-3: 0011 → 1100 → 1101

-4: 0100 → 1011 → 1100

① 4 − 3 = 1

```
  4     0 1 0 0   →   4
+ -3    1 1 0 1   →   2^4 - 3
-------------------------------
  1   | 0 0 0 1     2^4 + 1
```

② 3 − 4 = −1

```
  3     0 0 1 1   →   3
+ -4    1 1 0 0       2^4 - 4
-------------------------------
      | 1 1 1 1     2^4 - 1   ← neg.
```

0 0 0 0 flip

0 0 0 1 add 1

⇒ 1

⇒ −1

③ How about $4+4=8$

$$\begin{array}{r} 4 \quad 0100 \\ +\,4 \quad 0100 \\ \hline 1000 \end{array} \Rightarrow -8$$

$[-8, +7]$

Overflow Rules

① pos + pos must be pos.

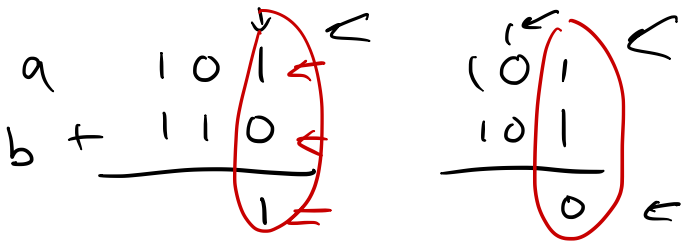
⇒ if two leading zeroes, but get leading 1 in answer

⇒ overflowed

② neg + neg must be neg.

⇒ if two leading 1s, but get leading 0 in answer

⇒ underflow.



a	b	answer
0	0	0
0	1	1
1	0	1
1	1	0

a	b	carry
0	0	0
0	1	0
1	0	0
1	1	1