

Last time

- Binary representations
- Conversions
 - four methods
 - examples
- Addition & subtraction
- Magic trick

Today

- Counting
- Powers of 2
 - Kilo, mega, giga, tera
 - bytes, nibbles
- Hexadecimal / Octal
 - examples: networking
- Negative numbers
 - two's complement

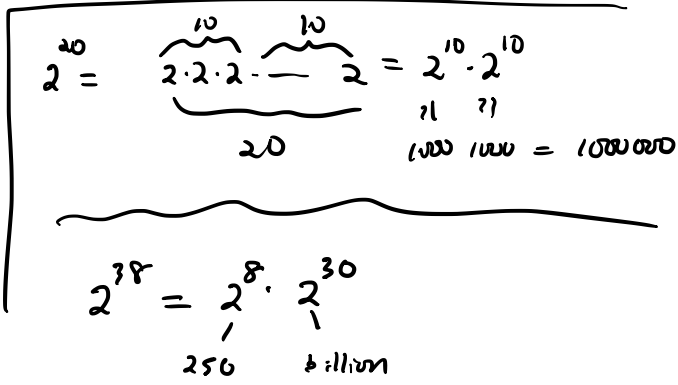
Next time

- Finish 2's comp.
- Implementing binary ops:
 - transistors, switches, logic gates

0 0000
 1 0001
 2 0010
 3 0011
 4 0100
 5 0101
 6 0110
 7 0111
 8 1000
 84?

$2^0 = 1$
 $2^1 = 2$
 $2^2 = 4$
 $2^3 = 8$
 $2^4 = 16$
 $2^5 = 32$
 $2^6 = 64$
 $2^7 = 128$
 $2^8 = 256$
 $2^9 = 512$
 $2^{10} = 1,024$

$2^{10} = \text{"kilo"} = K \quad 1024 \approx 1,000$
 $2^{20} = \text{"mega"} = M \quad 1,048,576 \approx 1,000,000$
 $2^{30} = \text{"giga"} = G \quad \sim 1,000,000,000$
 $2^{40} = \text{"tera"} = T \quad \sim 1,000,000,000,000$



byte: 8-bits

nibble: 4-bits

$$2^8 \cdot 2^7 \dots 2^0 = 2^8 - 1 = 255$$

$\underbrace{1111}_{2^8} \quad \underbrace{1111}_{2^7} \dots 2^0$

Other Representations: Octal, Hexadecimal

- Octal:
- Base 8 representation
 - Binary 3-bits at a time

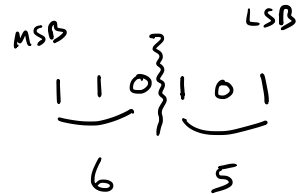
ex.

$$53_{10}$$

$$\begin{array}{r} 53 \\ - 48 \\ \hline 5 \end{array}$$

$6 \times 8 = 48$

$$\begin{array}{r} 64 \quad 8 \quad 1 \\ 6 \quad 5_8 \end{array}$$



Hexadecimal:

- Base 16 representation
- binary 4-bits at a time

53₁₀

256 16 1
3 5₁₆

} 0011 0101
3 5

53
48
—
5

47₁₀

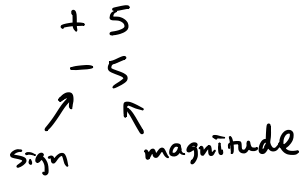
256 16 1
2 F₁₆

47
-32
—
15

10 11 12 13 14 15
0 1 2 3 -- 9 A B C D E F

Negative numbers: Signed magnitude, one's comp., two's comp.

Signed-mag:



sign 4 2 1

4-bit signed mag. rep.

+ 5: 0 1 0 1

- 5: 1 1 0 1

2 "zeros"

+ 0: 0 0 0 0

- 0: 1 0 0 0

1's complement

4-bit 1's comp.

① Positive #'s → represented in straight binary

$$5_{10} \rightarrow 0101$$

② Negative #'s → write magnitude in binary & flip all bits.

⇒ MSB will be sign

$$\begin{array}{l} -5_{10} \rightarrow \text{mag: } 0101 \\ \text{flip: } \textcircled{1010} \end{array}$$

$$\begin{array}{l} -7 : \text{mag: } 0111 \\ \text{flip: } 1000 \end{array}$$

$$+0 : 0000$$

$$-0 : \text{mag: } 0000$$

$$\text{flip: } 1111$$

Observation:

what is "flip all bits"?

$$\begin{array}{r} 16 \ 8 \ 4 \ 2 \ 1 \\ 1 \ 1 \ 1 \ 1 \end{array} \rightarrow 2^4 - 1$$

$$\begin{array}{r} \text{mag: } -0 \ 1 \ 0 \ 1 \\ \hline -5 \quad 1 \ 0 \ 1 \ 0 \end{array}$$

neg #'s in 1's comp: $-x : (2^4 - 1) - x \rightarrow \text{rep. in bin.}$

$$15 - 5 = 10_{10} \equiv 1010_2$$

$$x = 5$$

2's complement:

pos #'s \rightarrow rep. in bin. $+5 \Rightarrow 0101$

neg #'s \rightarrow rep. mag. in bin $-5 \Rightarrow 0101$

• flip all bits 1010

• add 1 $\textcircled{1011}$

$$\equiv 2^4 - x$$

$$16 - 5 = 11_{10} \equiv 1011_2$$

2's complement : subtraction is addition

① $4 - 3_{10} = 1$

8 4 2 1

$$\begin{array}{r} 4 : \quad 0100 \\ + -3 : \quad 1101 \\ \hline 1 \quad 1001 \end{array}$$

$$\begin{array}{r} 0011 \\ 1100 \\ 1101 \end{array}$$

$3 - 4 = -1$

$$\begin{array}{r} 3 : \quad 0011 \\ + -4 : \quad 1100 \\ \hline 1111 \end{array}$$

$$\begin{array}{r} 0100 \\ 1011 \\ 1100 \end{array}$$

←

-1 :

$$\begin{array}{r} 0001 \\ 1110 \\ \hline 1111 \end{array}$$