Last time

- Binary representations
- Conversions
  - four methods
  - examples
- Addition & subtraction
- Magic trick

Today

- Counting
- Powers of 2
  - Kilo, mega, giga, tera
  - bytes, nibbles
- Hexadecimal / octal
  - examples: networking
- Negative numbers
  - two's complement

Next time

- Finish 2's comp.
- Implementing binary ops:
  - transistors, switches, logic gates

0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
	8 4 2 1

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

$2^9 = 512$

$2^{10} = 1,024$

2^{10} = "kilo" = K   1024 $\approx$ 1,000

2^{20} = "mega" = M   1,048,576 $\approx$ 1,000,000

2^{30} = "giga" = G $\sim$ 1,000,000,000

2^{40} = "tera" = T $\sim$ 1,000,000,000,000

$$2^{20} = \underbrace{\overbrace{2\cdot 2\cdot 2}^{10}\cdot \text{—}\ \overbrace{\quad 2}^{10}}_{20} = \underset{\approx}{2^{10}}\cdot \underset{\approx}{2^{10}}$$

1000 1000 = 1000000

$$2^{38} = \underset{256}{2^8}\cdot \underset{\text{billion}}{2^{30}}$$

byte: 8-bits

nibble: 4-bits

$$\overset{2^8\ \cdot\ 2^7\ \text{—}\ \cdots\ 2^1\ 2^0}{\underbrace{1111}\ \underbrace{1111}} = 2^8 - 1 = 255$$

Other Representations: Octal, Hexadecimal

Octal:
- Base 8 representation
- Binary 3-bits at a time

ex. 53_{10}

$6 \times 8 = 48$

$$\begin{array}{r} 53 \\ -\ 48 \\ \hline 5 \end{array}$$

64	8	1
	6	5_8

MSB LSB

$$\underbrace{1\ 1\ 0}_{6}\ \Big\}\ \underbrace{1\ 0\ 1}_{5}$$

Hexadecimal:
- Base 16 representation
- binary 4-bits at a time

53_{10}

256	16	1
	3	5_{16}

0011 0101

3 5

$$\begin{array}{r} 53 \\ 48 \\ \hline 5 \end{array}$$

47_{10}

256	16	1
	2	F_{16}

$$\begin{array}{r} 47 \\ -32 \\ \hline 15 \end{array}$$

					10	11	12	13	14	15
0	1	2	3	-- 9	A	B	C	D	E	F

Negative numbers : Signed magnitude, one's comp. , two's comp.

Signed-mag:

+ 5

− 5

sign magnitude

4-bit signed mag. rep.

	sign	4	2	1
+ 5 :	0	1	0	1
− 5 :	1	1	0	1
+ 0 :	0	0	0	0
− 0 :	1	0	0	0

2 "zeroes"

1's complement

4-bit 1's comp.

① Positive #'s → represented in straight binary

$5_{10} \rightarrow$ 0101

② Negative #'s → write <u>magnitude</u> in binary & flip all bits.

⇒ MSB will be sign

$-5_{10} \rightarrow$ mag: 0101

flip: 1010

-7 : mag: 0111

flip : 1000

+0 : 0000

−0 : mag: 0000

flip: 1111

$$\begin{array}{rcccc} \underline{16} & 8 & 4 & 2 & 1 \\ & 1 & 1 & 1 & 1 & \rightarrow 2^4 - 1 \end{array}$$

Observation:

what is "flip all bits"?

mag: $-\ 0\ 1\ 0\ 1$

$-5 \quad 1\ 0\ 1\ 0$

neg #'s in 1's comp: $-x \ : \ \underbrace{(2^4-1)}\ -x \rightarrow$ rep. in bin.

$x = 5 \qquad 15 - 5 = 10_{10} \equiv 1010_2$

2's complement:

pos #'s $\rightarrow$ rep. in bin. $\quad +5 \Rightarrow 0101$

neg #'s $\rightarrow$

- rep. mag. in bin $\quad -5 \Rightarrow 0101$
- flip all bits $\quad 1010$
- add 1 $\quad 1011$

$\equiv 2^4 - x$

$16 - 5 = 11_{10} \equiv 1011_2$

2's complement : subtraction is addition

① $4-3_{10} = 1$

$$\begin{array}{rrr} 4 & : & {}^{1}0100 \\ + \; -3 & : & + \; 1101 \\ \hline 1 & & 1\,|\,\underbrace{0001} \end{array}$$

8 4 2 1

0011
1100
1101

$3-4 = -1$

$$\begin{array}{rrr} 3 & : & 0011 \\ + \; -4 & : & + \; 1100 \\ \hline & & 1111 \end{array}$$

0100
1011
1100

-1 : 0001
1110
1111