CS1800
Discrete Structures
Fall 2017

Lecture 27
11/8/17

Last time

- Finish Bayes Law
- Start Markov Chains

Today

- Finish M.C.

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- Start Algorithmic Analysis

Next time

- Continue Algorithmic Analysis
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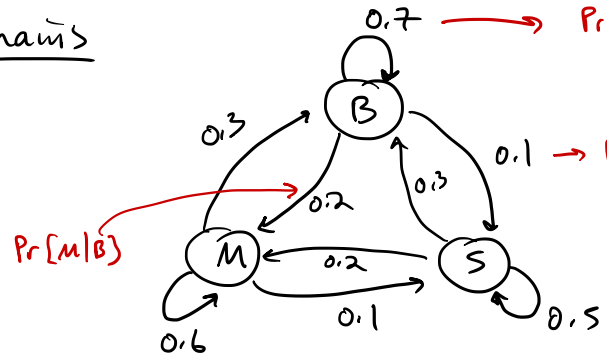
Markov Chains

B: Bertucci's
M: Margaritas
S: Sato



Pr[B|B]

Pr[S|B]

Pr[M|B]

3 reds ⇒ must add to 1

state
Transition Matrix

$$P = \begin{array}{c} \\ B \\ M \\ S \end{array} \begin{array}{c} \begin{array}{ccc} B & M & S \end{array} \\ \begin{pmatrix} .7 & .2 & .1 \\ .3 & .6 & .1 \\ .3 & .2 & .5 \end{pmatrix} \end{array}$$

Stochastic matrix

⇒ all rows sum to 1

$$B = 0.7 \cdot B + 0.3 \cdot M + 0.3 \cdot S$$

Stationary Distribution:
long-term fraction of time spent visiting each state.

B, M, S

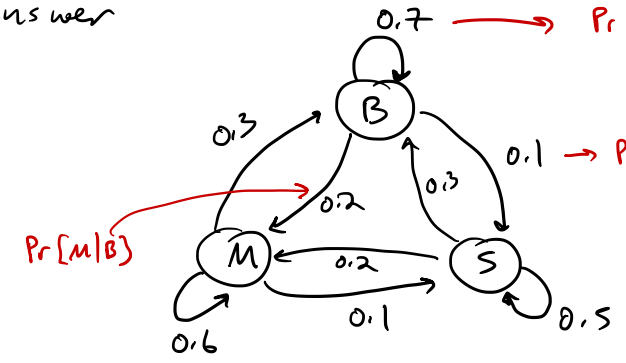
$$\vec{\pi} = \langle \pi_B, \pi_M, \pi_S \rangle$$

$$\pi_B + \pi_M + \pi_S = 1$$

Finding the s.d.:

1. Simulation
   - highly inefficient

② Create equations & solve for exact answer



① $B = .7B + .3M + .3S$

② $M = .2B + .6M + .2S$

③ $S = .1B + .1M + .5S$

But... I have one more equation:

④ $B + M + S = 1$

① $.3B - .3M - .3S = 0$

②+③ $.3B - .3M - .3S = 0$

$B + M + S = 1$

$M = .2B + .6M + .2S$

$S = .1B + .1M + .5S$

$\Rightarrow$

$B + M + S = 1$ ①

$-.2B + .4M - .2S = 0$ ②

$-.1B - .1M + .5S = 0$ ③

① $B + M + S = 1$

5×② $-B + 2M - S = 0$ $\Rightarrow$

10×③ $-B - M + 5S = 0$

$B + M + S = 1$

$3M = 1 \Rightarrow M = 1/3$

$6S = 1 \Rightarrow S = 1/6$

$\Rightarrow B = 1/2$

② Create equations & solve for exact answer

① $B = .7B + .3M + .3S$

② $M = .2B + .6M + .2S$

③ $S = .1B + .1M + .5S$

$\Rightarrow \quad \langle B, M, S \rangle = \langle B, M, S \rangle \begin{pmatrix} .7 & .2 & .1 \\ .3 & .6 & .1 \\ .3 & .2 & .5 \end{pmatrix}$

Guess: $B_0 = 1/3 \quad 1/2$

$M_0 = 1/3 \quad 1/3$

$S_0 = 1/3 \quad 1/6$

$\vec{\pi} = \vec{\pi} \cdot P$

$B_1 = .7 \cdot 1/3 + .3 \cdot 1/3 + .3 \cdot 1/3 = .433$

$M_1 = .2 \cdot 1/3 + .6 \cdot 1/3 + .2 \cdot 1/3 = .333$

$S_1 = .1 \cdot 1/3 + .1 \cdot 1/3 + .5 \cdot 1/3 = .233$

Algorithmic Analysis

n pages in dictionary

linear search : n

chunk search : $\frac{n}{c} + c \rightarrow 2\sqrt{n}$

c

bin. search : $\log_2 n$