CS1800
Discrete Structures
Fall 2017

Lecture 25
11/2/17

Last time

- Finish Expectation
  - variance / standard deviation
- Start Entropy

Today

- Finish Entropy
- Conditional Prob. & Bayes Law

Next time

- Markov Chains & Page Rank

Announcements

Regrades → Kevin Gold

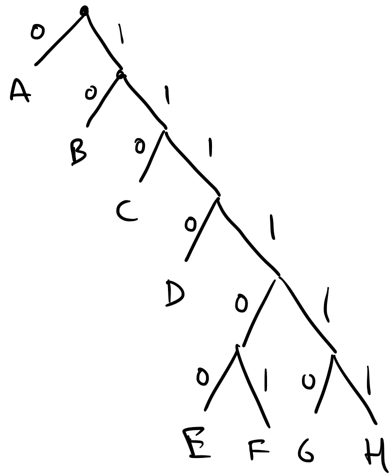
	A	B	C	D	E	F	G	H
$P_2 =$	$\frac{1}{2}$,	$\frac{1}{4}$,	$\frac{1}{8}$,	$\frac{1}{16}$,	$\frac{1}{64}$,	$\frac{1}{64}$,	$\frac{1}{64}$,	$\frac{1}{64}$
	0	10	110	1110	111100	111101	111110	111111

$$BPC = \sum_i l_i \cdot p_i$$

$$= 1 \cdot \tfrac{1}{2} + 2 \cdot \tfrac{1}{4} + 3 \cdot \tfrac{1}{8} + 4 \cdot \tfrac{1}{16}$$

$$+ 6 \cdot \tfrac{1}{64} \cdots + 6 \cdot \tfrac{1}{64}$$

$$= 2$$



p_i	l_i
$\frac{1}{2}$	1
$\frac{1}{4}$	2
$\frac{1}{8}$	3
$\frac{1}{16}$	4
$\frac{1}{64}$	6

$$p_i = \tfrac{1}{2}^k, \quad l_i = k \implies p_i = \tfrac{1}{2}^{l_i}$$

$$\iff 2^{l_i} = \frac{1}{p_i}$$

$$\iff \boxed{l_i = \log_2 \frac{1}{p_i}}$$

Entropy:-

Q: $P = (p_1, p_2, p_3, \ldots)$

$$l_i = \log_2 \frac{1}{p_i}$$

What is expected code length?

$$BPC = \sum_i p_i \cdot l_i$$

$$= \sum_i p_i \cdot \log_2 \frac{1}{p_i}$$

entropy

$$H(p) = \sum_i p_i \cdot \log_2 \frac{1}{p_i}$$

ACBADAFGAH

$p_A = \frac{4}{10}$

$p_B = 1/10$

$p_C = 1/10$

$p_D = 1/10$

$\vdots$

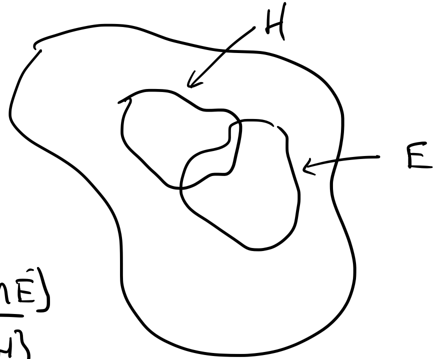
Facts

1. Entropy is the compression limit.
2. Entropy is essential to bound an ability to communicate in presence of noise.
3. Fundamental measure of randomness.
4. Cryptography

Conditional Probability

prob of E given H

$$\Pr\{H \cap E\} = \Pr\{H\} \cdot \underline{\Pr\{E \mid H\}}$$



$$\Rightarrow \Pr\{E \mid H\} = \frac{\Pr\{H \wedge E\}}{\Pr\{H\}}$$

$$= \Pr\{E\} \cdot \Pr\{H \mid E\}$$

$$\Rightarrow \Pr\{H \mid E\} = \frac{\Pr\{H \wedge E\}}{\Pr\{E\}}$$

Bayes Law:

$$\underline{\Pr\{E\}} \cdot \underline{\Pr\{H \mid E\}} = \Pr\{H \wedge E\} = \underline{\Pr\{H\}} \cdot \underline{\Pr\{E \mid H\}}$$

$$\Pr\{H \mid E\} = \frac{\Pr\{E \mid H\} \cdot \Pr\{H\}}{\Pr\{E\}}$$

H = hypothesis: do have Zika?

E = evidence: did blood test come back pos?