CS1800
Discrete Structures
Fall 2017

Lecture 23
10/30/17

Last time

- Finished Probability Examples
- Started Expectation

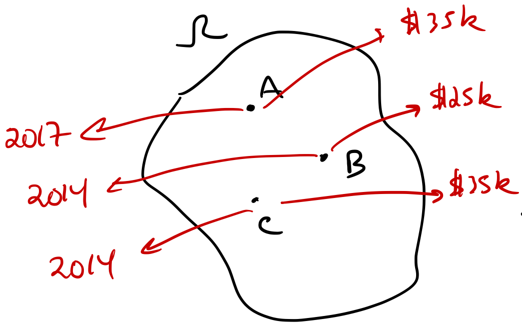
Today

- Expectation & Variance

Next time

- Entropy

Underlying probability measure induces distribution over range of rand. var.



- A – Alice's car
  - $35,000
  - 2017
- B – Bob's car
  - $25,000
  - 2014
- C – Charlie's
  - $35,000
  - 2014

$p(A) = p(B) = p(C) = 1/3$

probability measure

X_1 = purchase price

$\text{range}(X_1) = \{25000, 35000\}$

ind. dist. $D(25000) = \Pr\{X_1 = 25000\} = 1/3$

$D(35000) = \Pr\{X_1 = 35000\} = 2/3$

$$E\{X_1\} = \sum_x x \cdot \Pr\{X_1 = x\}$$

$$= 25000 \cdot 1/3 + 35000 \cdot 2/3$$

$$= 31666.67$$

X_2 = model year

$\text{range}(X_2) = \{2014, 2017\}$

$D(2014) = \Pr\{X_2 = 2014\} = 2/3$

$D(2017) = \Pr\{X_2 = 2017\} = 1/3$

$$E\{X_2\} = \sum_x x \cdot \Pr\{X_2 = x\}$$

$$= 2014 \cdot 2/3 + 2017 \cdot 1/3$$

$$= 2015$$

$$E(X_1) = \sum_{\omega \in \Omega} X_1(\omega) \cdot p(\omega)$$

$$= X_1(A) \cdot p(A) + X_1(B) \cdot p(B) + X_1(C) \cdot p(C) = 35000 \cdot 1/3 + 25000 \cdot 1/3 + 35000 \cdot 1/3 = 31666.67$$

(1) · Roll one fair six-sided die $\{⚀, ⚁, ⚂, ⚃, ⚄, ⚅\}$

$X =$ value of die face

$$X(⚀) = 1$$
$$X(⚁) = 2$$
$$\vdots$$
$$X(⚅) = 6$$

What is $E[X]$?

prob. meas: $p(⚀) = p(⚁) = p(⚂) = \cdots = p(⚅) = 1/6$ ← prob. meas. dictacted by rand. exp.

$\Rightarrow \Pr\{X=1\} = \Pr\{X=2\} = \cdots = \Pr\{X=6\} = 1/6$ ← induced dist. over range of r.v.

$$E[X] = \sum_{x} x \cdot \Pr\{X=x\} = \sum_{i=1}^{6} i \cdot \Pr\{X=i\} = 1 \cdot \Pr\{X=1\} + 2 \cdot \Pr\{X=2\} + \cdots$$
$$= 1 \cdot 1/6 + 2 \cdot 1/6 + \cdots \; 6 \cdot 1/6$$
$$= 1/6 \cdot (1+2+3+4+5+6) = \boxed{3.5}$$

$$E[X] = \sum_{\omega \in \Omega} X(\omega) \cdot p(\omega) = X(⚀) \cdot 1/6 + X(⚁) \cdot 1/6 + X(⚂) \cdot 1/6 + \cdots + X(⚅) \cdot 1/6$$
$$= 1 \cdot 1/6 + 2 \cdot 1/6 + \cdots + 6 \cdot 1/6$$
$$= 3.5$$

② Roll two fair six-sided die

$X =$ sum of values of die faces

what is $E[X]$

die 2

X:

die 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

die1 = 3
die2 = 5
(3,5)

$p(-,-) = 1/36$

$$E[X] = \sum_{x} x \cdot \Pr[X = x]$$

$X = 2 \quad \Pr[X=2] = 1/36$

$X = 3 \quad \Pr[X=3] = 2/36$

$X = 4 \quad \Pr[X=4] = 3/36$

$\vdots$

$X = 12 \quad \Pr[X=12] = 1/36$

$$E[X] = \sum_{\omega \in \Omega} X(\omega) \cdot p(\omega)$$

$$= \sum_{\omega \in \Omega} X(\omega) \cdot 1/36 = 1/36 \cdot \sum_{\omega \in \Omega} X(\omega)$$

$$= \frac{1}{36}\left(1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \cdots + 1 \cdot 12\right)$$

$$E[X] = \sum_{x} x \cdot \Pr[X = x]$$

$$= 2 \cdot 1/36 + 3 \cdot 2/36 + 4 \cdot 3/36 + \cdots + 12 \cdot 1/36$$

$$= 7$$

Linearity of Expectation

$X_1 =$ first die value

$X_2 =$ second die value

$$E[X_1 + X_2] = E[X_1] + E[X_2]$$

$$= 3.5 + 3.5 = 7$$

③ · Pay \$6 to play
· Roll 2 6-sided die
· payout is sum of die faces
except if doubles, then 0.

	1	2	3	4	5	6
1	-6	-3	-2	-1	0	1
2	-3	-6	-1	0	1	2
3	-2	-1	-6	1	2	3
4	-1	0	1	-6	3	4
5	0	1	2	3	-6	5
6	1	2	3	4	5	-6

$X =$ winnings (profit)

$$E\{X\} = \sum_x x \cdot Pr\{X=x\}$$

$X = -6 \qquad Pr\{X=-6\} = 6/36 = 1/6$

$X = -3 \qquad Pr\{X=-3\} = 2/36 = 1/18$

$\vdots$

$X = +5 \qquad Pr\{X=5\} = 2/36 = 1/18$

$$E\{X\} = \sum_x x \cdot Pr(X=x)$$
$$= (-6)\cdot 6/36 + (-3)(2/36) + \cdots + 5(2/36)$$
$$= -0.16\overline{6}$$

lose 16.6 ¢ per play or about 2.8%

$$E\{X\} = \sum_{\omega\in\Omega} X(\omega)\cdot P(\omega)$$
$$= \sum_{\omega\in\Omega} X(\omega)\cdot 1/36$$
$$= 1/36 \cdot \sum_{\omega\in\Omega} X(\omega)$$
$$= \frac{1}{36}\cdot\left((-6)\cdot 6 + (-3)\cdot 2 + \cdots + 5\cdot 2\right)$$
$$= \frac{1}{36}\cdot(-6) = -0.16\overline{6}$$