

CS1800
Discrete Structures
Fall 2017

Lecture 22
10/26/17

Last time

- Start probability

Today

- Continue Probability
 - more examples



- Start expectation & variance

Next time

- Finish expectation & variance

Sampling w/ replacement: 25 balls, 15 red & 10 blue

⇒ Draw 3 balls, one at a time, put back between draws

$$\Omega = \{ (R1, R1, R1), (R1, R1, R2), \dots, (B10, B10, B10) \}$$

$$|\Omega| = 25^3$$

• $E_1 =$ all red $|E_1| = 15^3$

• $P(E_1) = \frac{|E_1|}{|\Omega|} = \frac{15^3}{25^3} = \left(\frac{15}{25}\right)^3 = \left(\frac{3}{5}\right)^3 = \frac{27}{125} \approx 21.6\%$

• $E_2 =$ 2 red, 1 blue $|E_2| = \binom{3}{1} \cdot 10 \cdot 15^2$

↑ which blue
↑ when did I draw blue?
↑ which reds

$$P(E_2) = \frac{|E_2|}{|\Omega|} = \frac{\binom{3}{1} \cdot 10 \cdot 15^2}{25^3}$$
$$= \frac{3 \cdot 2 \cdot 8 \cdot 3^2 \cdot 5^2}{5^3 \cdot 5^3} = \frac{54}{125} = 43.2\%$$

- Bit strings (Bytes)
- Rand. Exp.:
 - flip fair coin 8 times
 - heads \rightarrow output 1
 - tails \rightarrow output 0
- \Rightarrow generates a byte.

$$\Omega = \{ 00000000, 00000001, \dots, 11111111 \}$$

$$|\Omega| = 2^8$$

- $E_1 =$ byte has exactly 4 1s.

$$|E_1| = \binom{8}{4} = 70$$

\nearrow
pick positions
for 1s

(fixes positions for 0s)

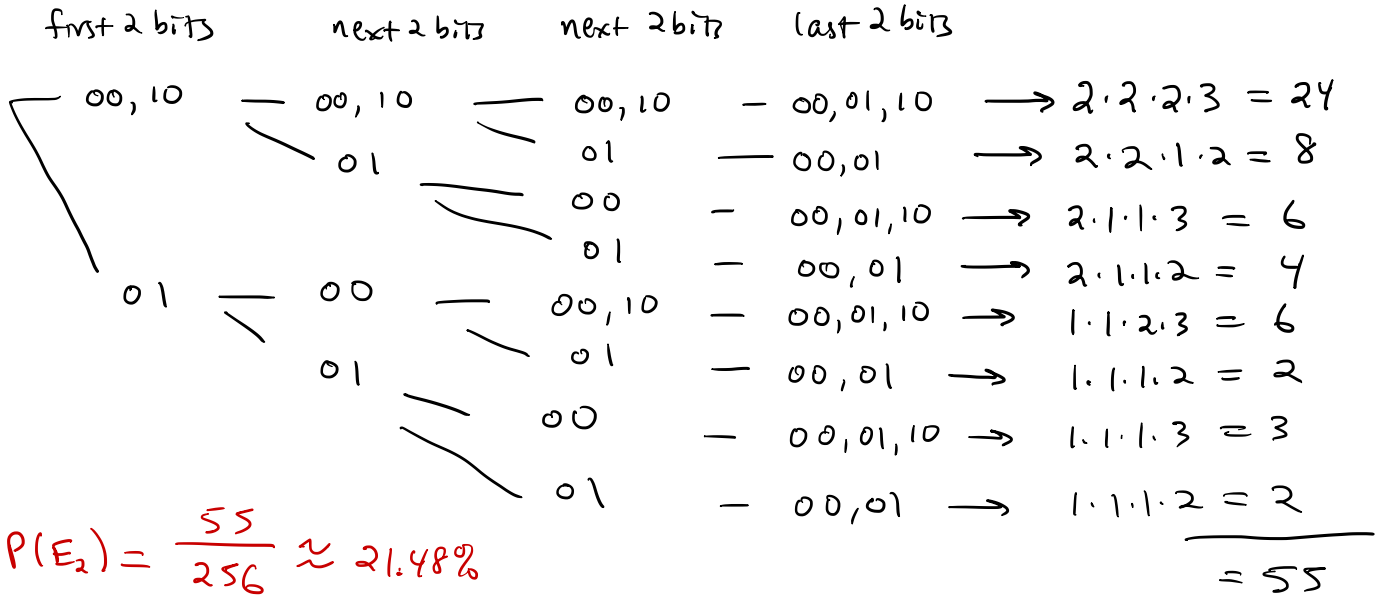
$$P(E_1) = \frac{70}{2^8} = \frac{70}{256} \approx 27.34\%$$

$E_2 =$ byte that does not contain consecutive 1s.

- generate 2 bits at a time $\left. \begin{matrix} 00 \\ 01 \\ 10 \\ \cancel{11} \end{matrix} \right\}$ allowed

- left-to-right

- if 01, then the next two bits can't be 10

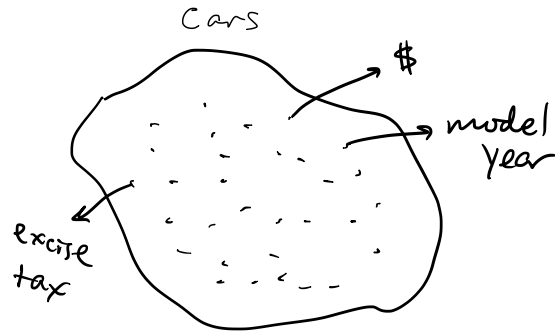


$$P(E_2) = \frac{55}{256} \approx 21.48\%$$

Expectation

- Random variable

$$X: \Omega \rightarrow \mathbb{R}$$



$$E[X] = \sum_x x \cdot \Pr[X=x]$$

↑
expected
or
"average"
value

$$E[X] = \sum_{\omega \in \Omega} X(\omega) \cdot p(\omega)$$

1/2 cars 27 000

1/3 cars 10 000

1/6 cars 15 000

$$E[X] = \sum_x x \cdot \Pr[X=x] = \frac{1}{2} \cdot 27000 + \frac{1}{3} \cdot 10000 + \frac{1}{6} \cdot 15000$$

-weighted average