CS1800
Discrete Structures
Fall 2017

Lecture 21
10/25/17

Last time	Today	Next time
• Finish counting – examples	• Start Probability	• Continue probability – examples

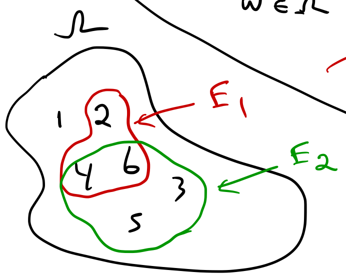
Probability

- Random experiment
- generates outcomes : $\omega \in \Omega$

  i.e. "omega"

  u.c. "omega"
- sample space : set of all possible outcomes Ω
- event : subset of sample space
- $p : \Omega \to \mathbb{R}$ probability measure
  - $0 \le p(\omega) \le 1 \quad \forall \omega \in \Omega$
  - $\sum_{\omega \in \Omega} p(\omega) = 1$

$\Rightarrow$ we will assume for now that $p(\omega) = \frac{1}{|\Omega|} \quad \forall \omega \in \Omega$



Example

- roll a fair six sided die
- rolled a 5
- $\{1, 2, 3, 4, 5, 6\} = \Omega$
- $E_1 = \text{"even"} = \{2, 4, 6\}$
- $E_2 = \text{"}\ge 3\text{"} = \{3, 4, 5, 6\}$
- $p(1) = p(2) = \cdots = p(6) = 1/6$

$$p(E) = \sum_{\omega \in E} p(\omega)$$

e.g. $p(E_1) = p(2) + p(4) + p(6)$

If $p(\omega) = \frac{1}{|\Omega|} \quad \forall \omega \in \Omega$

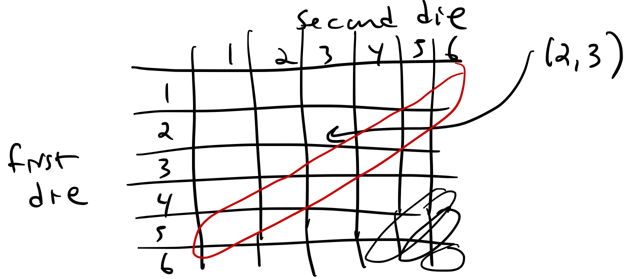
$$p(E) = \frac{|E|}{|\Omega|}$$

e.g. $p(E_1) = 3/6$

Examples

① Roll one fair die

$\Rightarrow \Omega = \{1,2,3,4,5,6\}$



② Roll two fair dice

$\Rightarrow \Omega = \{(1,1),(1,2),(1,3)\ldots,(2,1),(2,2)\ldots,(6,6)\} = \{1,2,\ldots 6\} \times \{1,2,\ldots 6\}$

$|\Omega| = 36$

$E_1 =$ total is 7

$E_1 = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$$p(E_1) = \frac{|E_1|}{|\Omega|} = \frac{6}{36} = \frac{1}{6}$$

$E_2 =$ total is greater than 8

$= 9$ or 10 or 11 or 12

$$|E_2| = \underset{12}{1} + \underset{11}{2} + \underset{10}{3} + \underset{9}{4}$$

$= 10$

$$p(E_2) = \frac{|E_2|}{|\Omega|} = \frac{10}{36} = \frac{5}{18}$$

Cards : Standard deck of cards : 4 suits Hearts, Diamonds, Clubs, Spades

within each suit $2, 3, 4, \ldots, 10, J, Q, K, A$

13 total per suit

- Rand. Exp. : draw one card from deck

- Sample space : $\Omega = \{2H, 3H, \ldots, AH, 2D, 3D, \ldots, AS\}$

$|\Omega| = 52$

- E_1 = face card (Jacks, Queen, King)

3 face cards per suit

4 suits

$|E_1| = 3 \cdot 4 = 12$

$$p(E_1) = \frac{|E_1|}{|\Omega|} = \frac{12}{52} = 3/13$$

- E_2 = card is between 2 & 10 (a number)

$|E_2| = 9 \cdot 4 = 36$

$$p(E_2) = \frac{|E_2|}{|\Omega|} = \frac{36}{52} = 9/13$$

Urn Problems :

15 red balls
10 blue balls

- Rand. Exp. : draw one ball from urn

- $\Omega = \{R1, R2, \ldots R15, B1, B2, \ldots B10\}$ $\quad |\Omega| = 25$

- E_1 = red $\quad |E_1| = 15$

$$P(E_1) = \frac{|E_1|}{|\Omega|} = \frac{15}{25} = 3/5$$

- Rand. exp. : draw 3 balls at once (sampling w/o replacement)

$\Omega = \{ \{R_1, R_2, R_3\}, \{R_1, R_2, R_4\}, \ldots \{R_1, R_2, B_1\} \ldots \{B8, B9, B10\} \}$

$|\Omega| = \binom{25}{3} = 2300$

- E_1 = all reds $\quad |E_1| = \binom{15}{3}$

$$P(E_1) = \frac{|E_1|}{|\Omega|} = \frac{\binom{15}{3}}{\binom{25}{3}} = \frac{455}{2300} = \frac{91}{460} \approx 19.8\%$$

- $E_2 =$ 2 red, 1 blue

$$|E_2| = \binom{15}{2} \cdot \binom{10}{1} = 1050$$

ways to get 2 reds

ways to get one blue

$$P(E_2) = \frac{\binom{15}{2}\binom{10}{1}}{\binom{25}{3}} = \frac{1050}{2300} \approx 45.7\%$$