CS1800
Discrete Structures
Fall 2017

Lecture 20
10/23/17

Last time

- Finish Perm. & Comb.

Today

- Example counting problems

Next time

- Probability

Telephone #s:

Former rules in US & Canada (617) 373-8168

Area codes: 3-digits First is not 0 or 1 312
Second must be 0 or 1

Exchange: 3-digits First & second not 0 or 1

Line #: 4-digits, not all zeroes.

How many: (area code: $8 \cdot 2 \cdot 10$) ($8 \cdot 8 \cdot 10$) (line #: $10^4 - 1$)

product product (all poss. − violations)

$= 1,023,897,600$ 781

Dinner Party: 16 people, circular table

Q: How many unique circular arrangements of people?

two ways to think about it

16! permutations, but many are rotations of each other

there are 16 rotations

A C D B

D B A C

$\Rightarrow \frac{16!}{16} = 15!$ circular arrangements

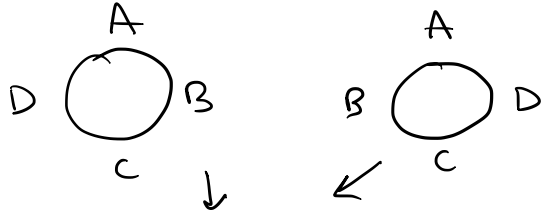
- sit first person down, any where
  - one way, since chairs indistinguishable in circular arrangement
- arrange the other 15 people around that person

15! ways

$\Rightarrow 1 \cdot 15! = 15!$

16 people - sitting around circular table

Q2: How many ways, where only consider people to left & right of guest?



guest	talk to
A	B, D
B	A, C
C	B, D
D	A, C

- every circular arrangement corresponds to a "talk to" list
- every "talk to" list gives rise to ... exactly 2 circular arrangements

$\Rightarrow$ $\frac{15!}{2}$ "talk to" lists

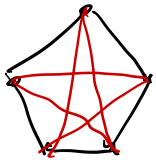
Dinner party, circular table, 16 guests, always same 16 people
- I attend. talk to people next to me

Q: How many parties until guaranteed that I talk to same person twice.

A:
- pigeonhole principle
- 15 people besides me → 15 slots
- each party - talk to 2 people
- at 8th party, will have talked to 16 people total

⇒ by PHP, must have talked to someone twice.

- Diagonals on an n-gon



pentagon or 5-gon

Q: How many diagonals?

- choose 2 different points $\rightarrow \binom{5}{2}$
- generate a line
- all are <u>diagonals</u> except 5 lines on outside

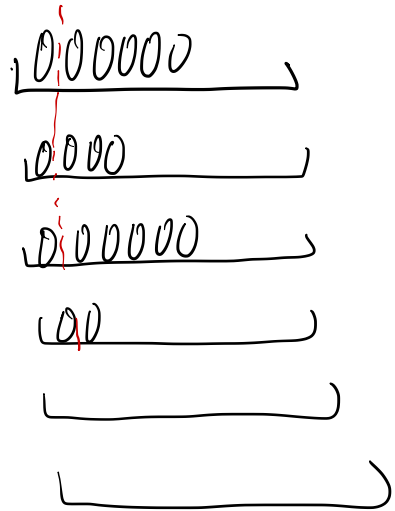
$$\Rightarrow \binom{5}{2} - 5 = \frac{5 \cdot 4}{2 \cdot 1} - 5 = 10 - 5 = 5$$

decagon: $\binom{10}{2} - 10 = \frac{10 \cdot 9}{2 \cdot 1} - 10 = 45 - 10 = 35$

n-gon: $\binom{n}{2} - n$

Books on shelves

- n books on k shelves
- alphabetical by author
- shelf capacity $\geq n$
- contiguous shelving
  - all empty shelves at bottom



Q: How many ways?

Choose, say, j shelves

$\Rightarrow$ each of j shelves must have a first book
- takes care of j books

$\Rightarrow$ $n-j$ books that remain, can be arranged w/ any amt
- balls - m - bins

$$\Rightarrow \binom{(n-j)+(j-1)}{j-1} = \binom{n-1}{j-1}$$

$$\sum_{1 \leq j \leq \min(k,n)} \binom{n-1}{j-1}$$