CS1800
Discrete Structures
Fall 2017

Lecture 18
10/18/17

Last time
- Pigeonhole Principle
- Permutations & Combinations

Today
- Finish Perm. & Comb.
  - examples
  - Balls-in-Bins
- Start Binomial Theorem

Next time
- Finish Bin. Thm.

Announcements : See Schedule page & Piazza

- Midterm Exam: Wed Oct 25 6-8pm
- Review Sessions

Sunday	1-2:40pm 3-4:40 pm	Paulu
Monday	6-8pm	Gold
Tuesday	6-8pm	Aslam

- All relevant lecture videos & notes will be posted

· Permutations r-Perm $P(n,r) = \#$ <u>ordered</u> subsets of r obj. from n

$$P(n,r) = \overbrace{n \cdot (n-1) \cdots (n-r+1)}^{r}$$
$$= \frac{n!}{(n-r)!}$$

$$P(n,n) = n!$$

· Combinations r-comb $C(n,r) = \binom{n}{r} = \#$ <u>unordered</u> subsets of r obj. from n.

$$P(n,r) = C(n,r) \cdot P(r,r)$$
$$= C(n,r) \cdot r!$$
$$\Rightarrow C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{r!\,(n-r)!}$$

Examples

10 package

6 heavy
4 light

- select 5 packages to deliver

$$\binom{10}{5} = \frac{10!}{5!\,(10-5)!} = \frac{10!}{5!\,5!} = 252$$

- select 3 heavy
  3 light to deliver

$$\binom{6}{3} \cdot \binom{4}{3} = 80$$

pick heavy　　pick light

E.g. 2

① How many bytes have exactly 3 1's ?

_ _ _ _ _ _ _ _

↑ pick (choose) positions for 3 1's.

$$C(8,3) = \binom{8}{3} = \frac{8!}{3! \cdot 5!} = \frac{8 \cdot 7 \cdot \cancel{6}}{\cancel{3 \cdot 2} \cdot 1} = 56$$

② How many bytes have exactly 5 1's?

$$C(8,5) = \binom{8}{5} = \frac{8!}{5! \cdot 3!} = 56$$

Why same?

Ⓐ ① picking 3 positions for 1's

② picking 5 positions for 1's

≡ picking 3 positions for 0's.

Ⓑ $$\binom{n}{r} = \frac{n!}{r! \cdot (n-r)!} = \frac{n!}{(n-r)! \cdot r!} = \binom{n}{n-r}$$

E.g. Password Space :

- 6 to 10 characters long
- at least 3 letters
- at least 3 digits

— u.c. letters
— l.c. letters
— digits

How many?

- First decision: length 6, 7, 8, 9, 10 — sum rule, no inc./exc.
- Once pick length (e.g. 9), count # legal passwords of that length.
  - Count # total passwords of length 9 $= 62^9$
  - Subtract # illegal passwords of length 9.

pick position for letter

Pick letter

pick remaining digits

sum rule

non-overlapping

no inc/exc.

0	letters	—	10^9
1	letter	—	$9 \cdot 52 \cdot 10^8$
2	letters	—	$\binom{9}{2} \cdot 52^2 \cdot 10^7$
0	digits	—	52^9
1	digit	—	$9 \cdot 10 \cdot 52^8$
2	digits	—	$\binom{9}{2} \cdot 10^2 \cdot 52^7$

$$62^9 - \left(10^9 + 9 \cdot 52 \cdot 10^8 + \cdots + \binom{9}{2} \cdot 10^2 \cdot 52^7 \right)$$

Balls-in-bins

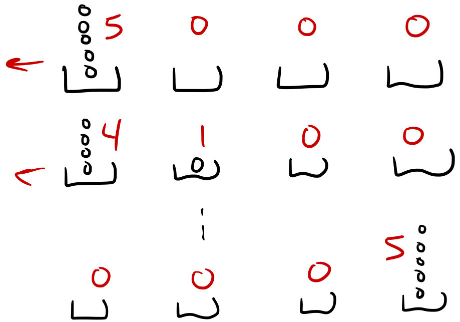
$S = \{a, b, c, d\}$

$\{c, d\} = 0011$

E.g.
- 4 bins
- 5 ball (indistinguishable)
- Q: How many ways to place those 5 balls into 4 bins?

00000111

00001011



algebraicly...

$$x_1 + x_2 + x_3 + x_4 = 5$$

where each $x_i \in \mathbb{N}$

$\Rightarrow$ $00|0|0|0$ $\Rightarrow$ 00101010 $\binom{8}{3}$

arrangements of n balls into k bins IS

$$\binom{n+k-1}{k-1} = \binom{n+k-1}{n}$$

$$\binom{n}{r} = \binom{n}{n-r}$$

Q: 5 brands of soda, buy 15 cans

5	3	4	2	1
coke	pepsi	MD	sprite	DP

$$\binom{15+(5-1)}{5-1} = \binom{19}{4} = 3876$$