CS1800
Discrete Structures
Fall 2017

Lecture 16
10/12/17

Last time

- sets
- set operations

Today

- Finish set operations
- Computer representations of sets
- Basic rules for counting

Next time

- Permutations & Combinations

Power Set

$P(A) =$ set of all subsets of A

$$A = \{\overset{1}{a}, \overset{1}{b}, \overset{0}{c}\}$$

$$P(A) = \{\overset{000}{\emptyset}, \overset{100}{\{a\}}, \overset{010}{\{b\}}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$$

$$|P(A)| = 2^{|A|}$$

a	b	c	
0	0	0	$\emptyset$
0	0	1	$\{c\}$
0	1	0	$\{b\}$
0	1	1	$\{b,c\}$
⋮			
1	1	1	$\{a,b,c\}$

$$P(\emptyset) = \{\emptyset\}$$

$$|\emptyset| = 0$$

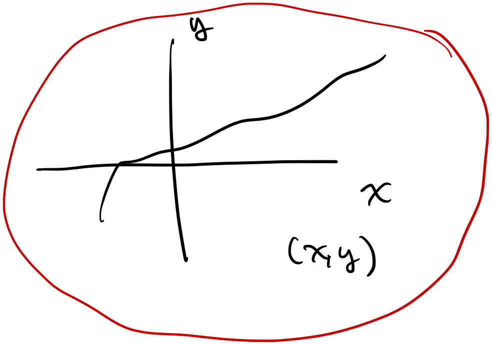
$$|P(\emptyset)| = 1$$

$$|P(A)| = 2^{|A|}$$

$$|P(\emptyset)| = 2^{|\emptyset|} = 2^0 = 1$$

Cartesian Products

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x,y) \mid x \in \mathbb{R},\ y \in \mathbb{R}\}$$



$$A \times B = \{(x,y) \mid x \in A,\ y \in B\}$$

e.g. $A = \{1,2,3\}$

$B = \{a,b\}$

$$(1,a) \neq (a,1)$$

$$A \times B = \{(1,a), (2,a), (3,a), (1,b), (2,b), (3,b)\}$$

$$B \times A = \{(a,1), (a,2), \text{——} (b,3)\}$$

$$A \times B \neq B \times A$$

b	(1,b)	(2,b)	(3,b)
a	(1,a)	(2,a)	(3,a)
	1	2	3

$$|A \times B| = |A| \times |B|$$

$$A \times B \times C = \{ (x, y, z) \mid x \in A,\ y \in B,\ z \in C)$$

$$| A \times B \times C | = |A| \times |B| \times |C|$$

$$| A_1 \times A_2 \times A_3 \times \dots \times A_n | = |A_1| \times |A_2| \times \dots \times |A_n|$$

$(a_1, a_2, \dots, a_n)$ - n-tuples

Representations of Sets

		1	2	3	4	5	6	7	8	9	10
	$U = \{1, 2, 3, \ldots, 10\}$	1	2	3	4	5	6	7	8	9	10
	$A = \{2, 4, 6, 8, 10\}$	0	1	0	1	0	1	0	1	0	1
	$B = \{1, 2, 3, 4, 5\}$	1	1	1	1	1	0	0	0	0	0
bitwise OR $\rightarrow$	$A \cup B$	1	1	1	1	1	1	0	1	0	1
bitwise AND $\rightarrow$	$A \cap B$	0	1	0	1	0	0	0	0	0	0
bitwise XOR $\rightarrow$	$A \Delta B$	1	0	1	0	1	1	0	1	0	1
bitwise NOT $\rightarrow$	$\overline{B}$	0	0	0	0	0	1	1	1	1	1

$$\neg (A \wedge B) = \neg A \vee \neg B$$

$$\overline{(A \cap B)} = \overline{A} \cup \overline{B}$$

Sum Rule & Product Rule

Silly example

- 8 pants, 6 shorts — pants <u>OR</u> shorts $\Rightarrow$ $14 = 8+6$
- 8 pants, 12 shirts — pants <u>AND</u> shirts $\Rightarrow$ $8 \times 12 = 96$

Product Rule: If A & B are finite sets, then the number of ways to pick an element from A <u>and</u> then an element from B is $|A| \times |B| = |A \times B|$

More generally... $A_1, A_2, \ldots A_n \rightarrow |A_1| \times |A_2| \times \cdots \times |A_n|$

Examples

- 4-char passwords, upper/lower case & digits

  $A = \{a, b, \ldots z$
  $A, B, \ldots Z,$
  $0, 1, 2, \ldots 9\}$

- how many?

  $A \times A \times A \times A$

$$62 \times 62 \times 62 \times 62 = 62^4 = 14{,}776{,}336$$

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Jay	Kevin	Virgil
407	79	103

3-person committee, one student per section

$$407 \times 79 \times 103 = 3{,}311{,}759$$

Sum Rule

If A and B are disjoint finite sets, then the number of ways of picking an object from A or B is $|A|+|B|$

Generally $A_1, A_2, \ldots, A_n$ are all mutually disjoint

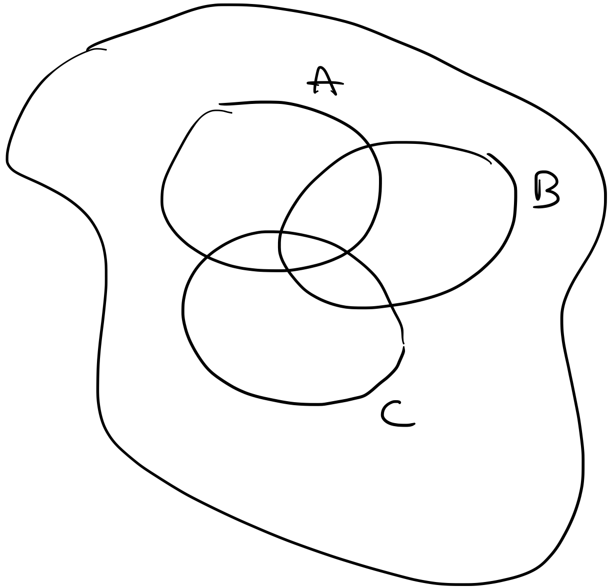
then $|A_1 \cup A_2 \cdots \cup A_n| = |A_1| + |A_2| + \cdots + |A_n|$

Principle of Inclusion-Exclusion

2 sets A & B

$$|A \cup B| = |A| + |B| - |A \cap B|$$

3 sets A, B, C



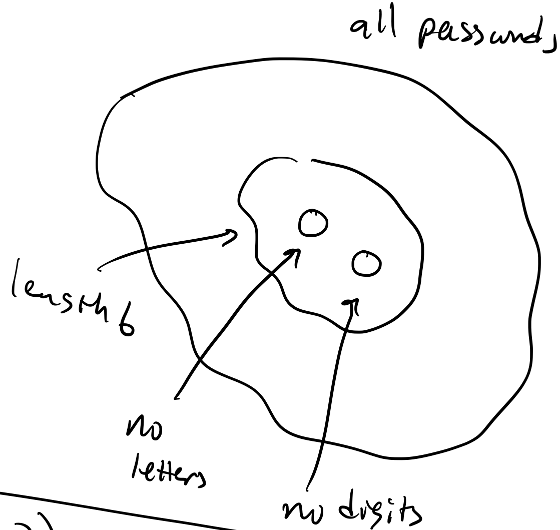
$$|A \cup B \cup C| = |A| + |B| + |C|$$
$$- |A \cap B| - |A \cap C| - |B \cap C|$$
$$+ |A \cap B \cap C|$$

Example: Passwords between 6 and 10 chars, upper/lower case letters or digits, at least one digit and at least one letter

How many?

- pick length 6 or 7 or 8 or .. or 10
- mutually disjoint → apply sum rule

focus on length 6 passwords.



all − illegal

$62^6 - (10^6 + 52^6)$

$62^6 - 10^6 - 52^6$

total: $\boxed{(62^6 - 10^6 - 52^6) + (62^7 - 10^7 - 52^7) + \cdots + (62^{10} - 10^{10} - 52^{10})}$