

CS1800
Discrete Structures
Fall 2017

Lecture 15
10/11/17

Last time

Finish RSA

Today

- Module 3: Counting/combinatorics
 - Sets & set operations

Next time

- Basic rules for counting

Motivating Example: Password Spaces

4-digit PIN: $10,000 = 10^4$

4-lower case chars: $26^4 = 456,976$

4-upper or lower case chars: $52^4 = 7,311,616$

NSF passwords: 7 to 10 chars
upper, lower case letters
digits
 ≥ 2 letters
 ≥ 2 digits

Sets : $S_1 = \{0000, 0001, 0002, \dots, 9999\}$ 4-digit PINs

$S_2 = \{\text{red, blue, green, yellow}\}$

$= \{\text{blue, green, yellow, red}\}$

Common sets:

$\mathbb{N} = \{0, 1, 2, 3, \dots\}$ natural numbers

$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ integers

$\mathbb{Z}^+ = \{1, 2, 3, \dots\}$ positive integers

$\mathbb{Q} = \{p/q \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0\}$ rationals

$\mathbb{R} =$ real numbers

$\{v \mid \text{conditions on } v\}$

$\mathbb{Z}_n = \{0, 1, 2, \dots, n-1\}$ int. mod n

$\emptyset = \{\}$ empty set

Set Builder Notation

$$S = \{v \mid \text{conditions on } v\}$$

e.g. $S = \{x \mid x \in \mathbb{Z}, |x| < 5\}$

what is S ? $S = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$$3 \in S$$

$$7 \notin S$$

"element of"

"not element of"

Cardinality : size of set

$$\begin{aligned} |S| &= \text{"size" of set } S \\ &= 9 \end{aligned}$$

cardinalities can be finite - e.g. $|S| = 9$

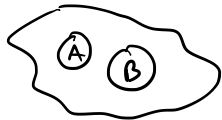
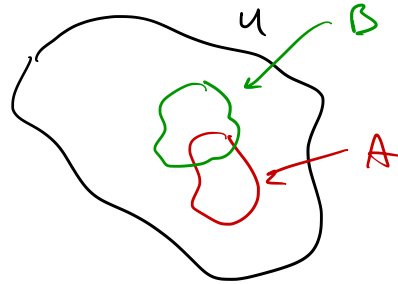
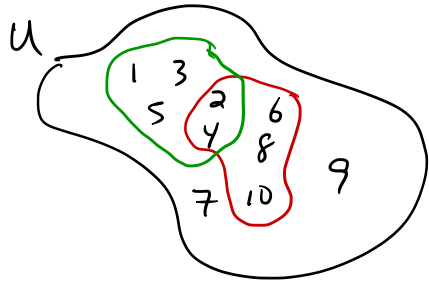
or infinite - e.g. $|\mathbb{N}|, |\mathbb{Z}|, |\mathbb{Q}|, |\mathbb{R}|$

Venn Diagram

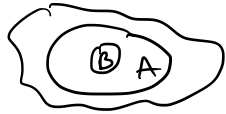
$$U = \{1, 2, 3, \dots, 10\}$$

$$A = \text{"evens"} = \{x \mid x \in U, x \text{ is even}\} = \{2, 4, 6, 8, 10\}$$

$$B = \text{"}\leq 5\text{"} = \{x \mid x \in U, x \leq 5\} = \{1, 2, 3, 4, 5\}$$



Def. A & B are disjoint if A & B share no elements in common, i.e. they have empty intersection.

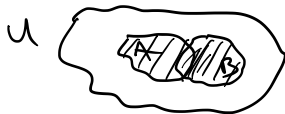


Def. B is a subset of A if every element of B is an element of A . $B \subseteq A$

$$A \subseteq A$$

Def. Proper subset $B \subset A$: $B \subseteq A$ and
A contains other elements

Set Operations



$$U = \{1, 2, \dots, 10\}$$

$$A = \{2, 4, 6, 8, 10\}$$

$$B = \{1, 2, 3, 4, 5\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$$

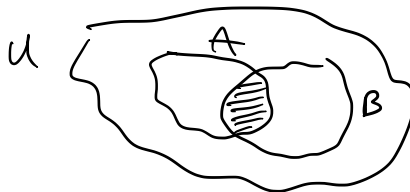
① Unions

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

② Intersection

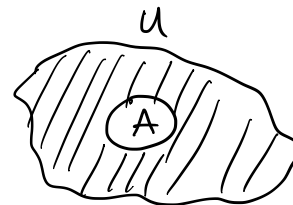
$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

$$A \cap B = \{2, 4\}$$



③ Complement

$$\overline{A} = \{x \mid x \in U \text{ and } x \notin A\}$$

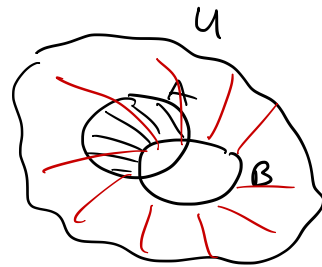


$$\overline{A} = \{1, 3, 5, 7, 9\}$$

$$\overline{B} = \{6, 7, 8, 9, 10\}$$

Set Difference

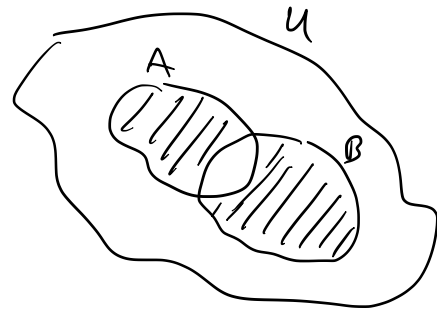
$$\begin{aligned} A - B &= \{ x \mid x \in A \text{ and } x \notin B \} \\ &= A \cap \overline{B} \end{aligned}$$



Symmetric Difference

$$\begin{aligned} A \Delta B &= \{ x \mid (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A) \} \\ &= (A - B) \cup (B - A) \\ &= (A \cap \overline{B}) \cup (B \cap \overline{A}) \end{aligned}$$

$$\begin{aligned} \text{also...} &= (A \cup B) - (A \cap B) \\ &= (A \cup B) \cap \overline{(A \cap B)} \end{aligned}$$



Power Set

$\mathcal{P}(A)$ = set of all subsets of A

$$A = \{a, b, c\}$$

$$\mathcal{P}(A) = \{ \overset{000}{\phi}, \overset{100}{\{a\}}, \overset{010}{\{b\}}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \}$$

$$|\mathcal{P}(A)| = 2^{|A|}$$