CS1800
Discrete Structures
Fall 2019

Lecture 15
10/29/19

Last time

- Finished probability
  - Birthday Paradox
- Mathematics of Algorithmic Analysis
  - Search algorithms

Today

- More motivation
  - sort algorithms
- Sequences
- Series

Next time

- Continue sequences & series

HW 6

- Problem 4

Sorting

Insertion Sort : $1+2+3+4+5+\cdots+n = \frac{n(n+1)}{2} = \frac{n^2}{2}+n/2$

Selection Sort : $n+(n-1)+(n-2)+\cdots+3+2+1 = \frac{n(n+1)}{2}$

Merge Sort :

6 3 7 2 | 4 8 1 5

$\cancel{2}$ 3 6 7 $\quad$ $\cancel{1}$ 4 5 8

1 2 3 4 5 6 7 8

6 3 7 2 | 4 8 1 5

6 3 | 7 2   4 8 | 1 5

6 | 3   7 | 2   4 | 8   1 | 5

splitting phase

3 6   2 7   4 8   1 5

2 3 6 7   1 4 5 8

1 2 3 4 5 6 7 8

merging

$$T(n) = 2 \cdot T(n/2) + n$$

answer:

$T(n) = n \log_2 n + n$

maybe

$T(n) = n^2$?

$$n^2 \stackrel{?}{=} 2 \cdot (n/2)^2 + n$$

$$= 2 \cdot \frac{n^2}{4} + n$$

$$= \frac{n^2}{2} + n$$

$n = 1,000,000 = 10^6$

I. S.	n^2	$\rightarrow$	$10^{12} = 1,000,000,000,000$	trillion
S. S.	n^2	$\rightarrow$	10^{12}	
M. S.	$n \log_2 n$	$\rightarrow$	$20,000,000$	20 million

$>$ factor of 50,000

1 sec vs. 13.88 hours

Sequences & Series

- Sequence: a_n : $a_1, a_2, a_3, \ldots$

- Examples

  - $1, 1/2, 1/3, 1/4, 1/5, \ldots$ $\qquad a_n = 1/n$

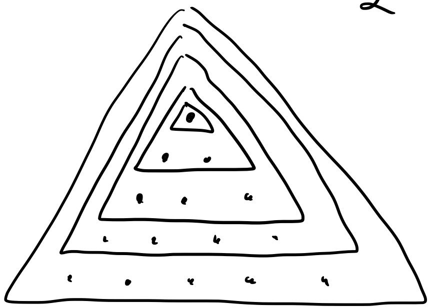
  - $2, 4, 6, 8, 10, \ldots$ $\qquad a_n = 2 \cdot n$

  - $7, 8, 9, 10, 11, \ldots$ $\qquad a_n = n + 6$

  - $1, 3, 6, 10, 15, 21, \ldots$ $\qquad a_n = \dfrac{n(n+1)}{2}$

    $\uparrow$

    triangular numbers



Common types of sequences: arithmetic, geometric, quadratic (harmonic)

① Arithmetic sequences ← difference between consecutive elements is constant.

1 2 3 4 5 ...

e.g. 4, 7, 10, 13, 16, ...

3 3 3 3

a

d

$a = 4$ (starting value)

$d = 3$ (diffence)

$$a_n = a + (n-1)\cdot d \implies a_n = 4 + (n-1)\cdot 3$$

$$= 3n + 1$$

starting value

constant difference

$$a_n = a + (n-1)\cdot d = d\cdot n + (a-d)$$

informative

simplified

E.g.

$$-7,\ -1,\ 5,\ 11,\ 17,\ 23,\ 29,\ \ldots$$

$$6\quad 6\quad 6\quad 6\quad 6\quad \ldots$$

$$a_n = -7 + (n-1)\cdot 6$$

$$= 6n - 13$$

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② Geometric Sequences ← constant factor between consecutive elements.

$$a_n = a \cdot r^{n-1}$$

↑ starting value

↖ constant ratio

e.g.

$$\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ 1, & 1/2, & 1/4, & 1/8, & 1/16, \ldots \end{array}$$

$$3,\ 6,\ 12,\ 24,\ 48,\ \ldots$$

$a = 3$

$r = 2$

$$a_n = 3\cdot 2^{n-1}$$

$$= \frac{3}{2}\cdot 2^n$$

$a = 1$

$r = 1/2$

$$a_n = 1\cdot (1/2)^{n-1}$$

$$= (1/2)^{n-1}$$

Quadratic sequences $\leftarrow$ second differences are constant

e.g.

$$\begin{array}{ccccccccccc} 1 & & 3 & & 6 & & 10 & & 15 & & 21 \quad \dots \\ & 2 & & 3 & & 4 & & 5 & & 6 & \\ & & 1 & & 1 & & 1 & & 1 & & \end{array}$$

$\leftarrow$ first differences

$\leftarrow$ second diffences

$a_n = a \cdot n^2 + b \cdot n + c$ for some a, b, c

$n=1 \qquad 1 = a \cdot 1^2 + b \cdot 1 + c \Rightarrow a + b + c = 1$

$n=2 \qquad 3 = a \cdot 2^2 + b \cdot 2 + c \Rightarrow 4a + 2b + c = 3$

$n=3 \qquad 6 = a \cdot 3^2 + b \cdot 3 + c \Rightarrow 9a + 3b + c = 6$

$> 3a + b = 2$

$> 5a + b = 3$

$> 2a = 1$

$\boxed{a = 1/2}$

$3 \cdot 1/2 + b = 2$

$\boxed{b = 1/2}$

$1/2 + 1/2 + c = 1$

$\boxed{c = 0}$

$$\Rightarrow a_n = \frac{1}{2} \cdot n^2 + \frac{1}{2} \cdot n + 0$$

$$= \frac{1}{2} n^2 + \frac{1}{2} n = \frac{1}{2}(n^2 + n)$$

$$= \frac{n(n+1)}{2}$$