

**DP**

Coins: Denom set  $D = \{d_1=1, d_2=5, d_3=10, d_4=25\}$

Give  $n$  cents change with min # coins.

$DP[n] = \text{min \# coins for } n \text{ cents.}$

$n = \text{input} \Rightarrow DP[n]$  is the PB.

$n=100 \quad DP[100] = 4$

$n=98 \quad DP[98] = 3 + 2 + 3 = 8$

Input  $n = 98$

PB = min # coins for  $n=98$

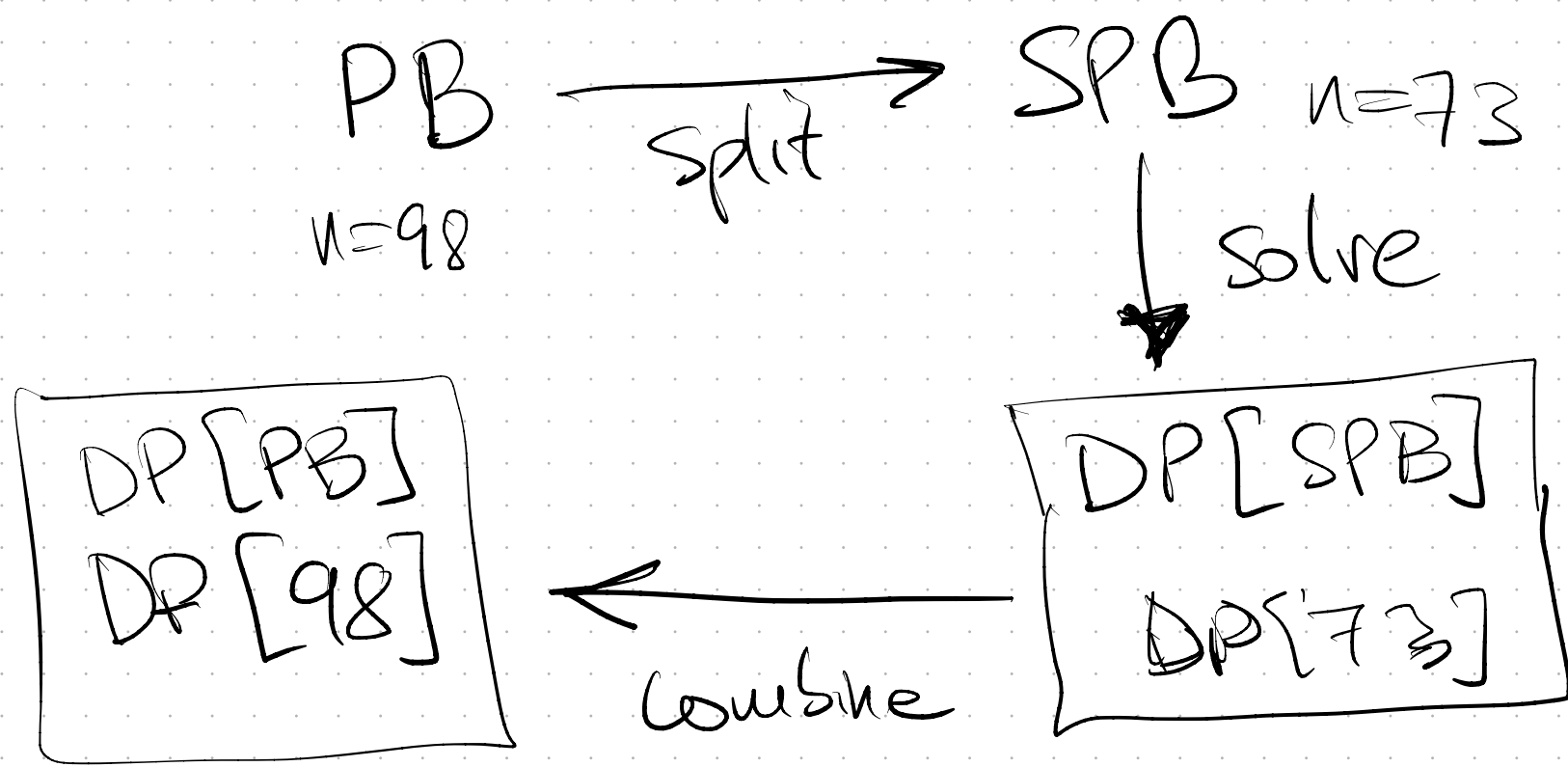
SPB = min # coins for  $m < n$

$DP[73] \Rightarrow$  SPB

min # coins for 73 cents.

SPB:  $DP[97] \quad DP[96] \quad DP[95]$

$DP[1]$



Greedy  
split by GC (proof)  
directly move  
onto resulting  
SPB

DP  
try all  
splits  $\Rightarrow$  all  
SPB  
and pick  
best

coin: "split" = which coin  
to use first

greedy: choose quarter 25c

$$DP[98] = 1 + DP[73]$$

quarter

DP

DP[98] possibilities

quora:  $1 + DP[73]$

dime:  $1 + DP[88]$

sc:  $1 + DP[93]$

...

...

MIN  $\left\{ \begin{array}{l} \text{quora} \\ \text{dime} \\ \text{sc} \end{array} \right.$

$$D = \{1, 7, 10\}$$

$n = 14$  greedy:  $10 + 1 + 1 + 1 + 1$  fails!

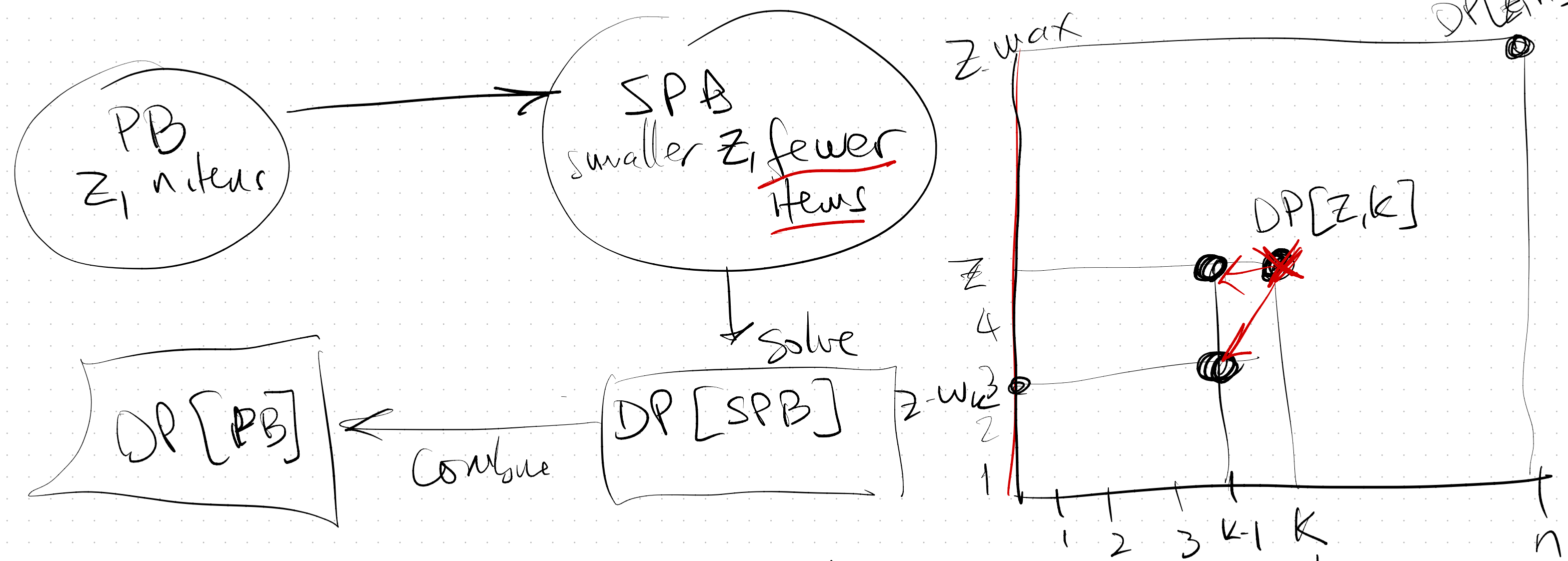
optimal:  $7 + 7$

$$\begin{aligned} DP[14] &= 1 + DP[4] \\ \text{MIN} \left\{ \begin{aligned} &= 1 + DP[7] \\ &= 1 + DP[13] \end{aligned} \right. \end{aligned}$$

*(Note: Red arrows in the original image point from DP[4] to DP[7] and from DP[7] to DP[13].)*

Knapsack  $n$  items weights  $w_1, w_2, \dots, w_n$  values  $v_1, v_2, \dots, v_n$   $Z = \text{max weight}$

$DP[Z, \text{items available } I_n = \{1, 2, 3, \dots, n\}] = \text{max value by selecting a subset of items in } I_n \text{ subj to } \sum \text{weights} \leq Z.$



$DP[Z, k] = \text{max val keep cap} = Z$   
 items avail =  $\{1, 2, 3, \dots, k\}$

$DP[Z, k]$   
 max value items  $\{1, 2, \dots, k\}$   
 total weight  $\leq Z$

MAX

- choose item  $k$  :  $v_k + DP[Z - w_k, k - 1]$   
 items  $1, 2, \dots, k-1$

- don't choose  
 item  $k$

$0 + DP[Z, k - 1]$

RT,  $n \times Z \times \Theta(1) = \Theta(nZ)$   
 comp per all

# Matrix Chain Mult.

$$A_1 A_2 A_3$$

$$(A_1 A_2) A_3 \quad (A_2 A_3) A_1$$

$A_1 A_2$  split

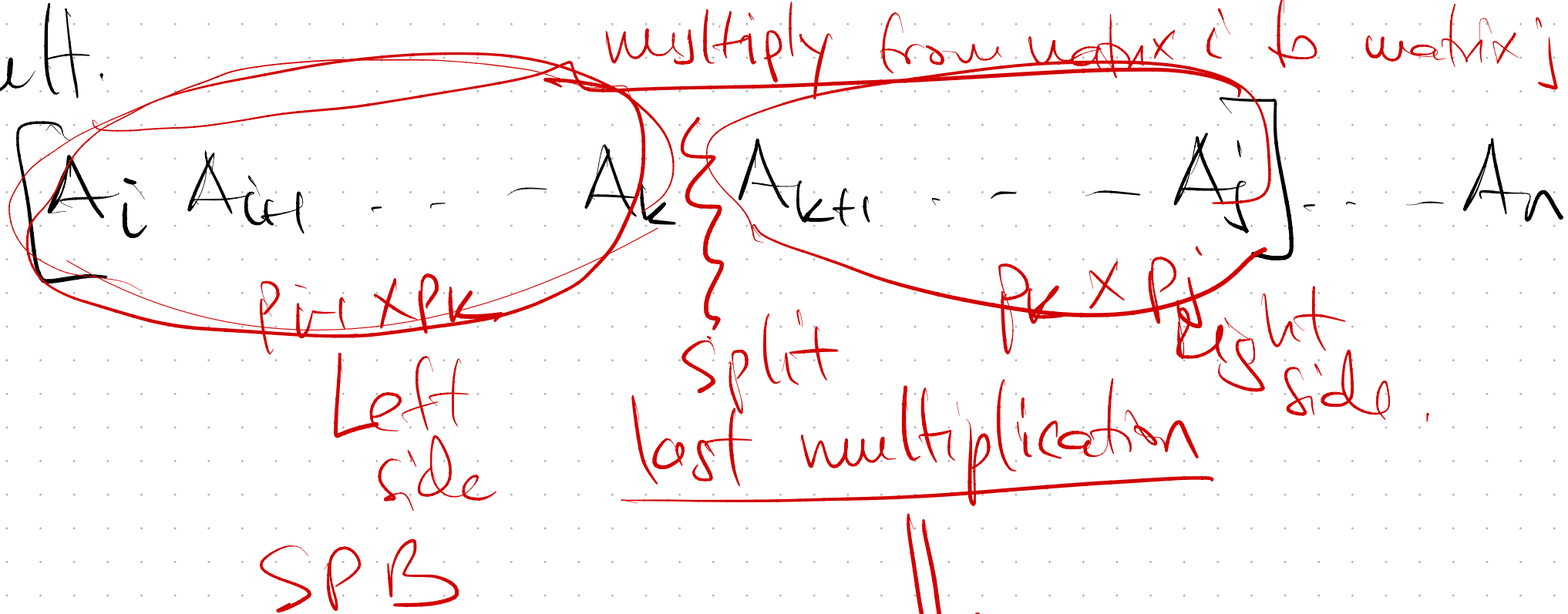
$A_4 A_5$

$A_3 (A_4 A_5)$

last:  $(A_1 A_2) \times (A_3 A_4 A_5)$

min  
 $i \leq k \leq j-1$

split  
 $K \rightarrow$



$$L_{SPB} \times R_{SPB}$$

$DP[i, j]$  = min # multiplications required to multiply  $A_i \cdot A_{i+1} \cdot \dots \cdot A_j$

Left:  $DP[i, k]$  + Right:  $DP[k+1, j]$  +  $P_{i-1} \times P_k \times P_j$

*last mult*

