

REC 10: Probabilities

Problem 1 cond-prob red black card

A set of 6 cards contains two black cards numbered 1 and 2 and four red cards numbered 1, 2, 3, 4. Each of the questions below refers to uniformly selecting **one** card from the set of 6.

Please be mindful that you justify your answer algebraically (e.g. for conditional probabilities, be sure to reference $P(A|B) = \frac{P(A,B)}{P(B)}$)

- i What is the probability that you draw a card whose value is 4?
- ii What is the probability that you draw a card whose value is 4 given the card is black?
- iii What is the probability that you draw a card whose value is 4 given the card is red?
- iv Consider how the probability of choosing a four changes between parts i and ii. Tell if the probability increased or decreased and provide a sentence or two which provides an intuitive understanding of this change.
- v Consider how the probability of choosing a four changes between parts i and iii. Tell if the probability increased or decreased and provide a sentence or two which provides an intuitive understanding of this change.

Problem 2 bayes gold prospecting

Gold prospecting is the search for new deposits of gold within the earth. If a parcel of land has a gold deposit, then there is a 90% chance that bits of gold will be found in one of its streams. However, gold can travel downstream into another parcel too! If a piece of land doesn't have a gold deposit, there is still a 40% chance that bits of gold are found in its streams. Only 30% of the parcels have gold deposits.

Let $D = 1$ be the event that a parcel has a gold deposit and $S = 1$ be the event that gold may be found in its streams.

- i Describe, in english, each of the probabilities below.
 - (a) $P(D = 1)$
 - (b) $P(D = 1|S = 1)$
 - (c) $P(S = 0|D = 1)$
- ii Compute $P(S = 1)$
- iii What is the probability that a parcel of land has a gold deposit given that no gold may be found in its streams?

Problem 3 count tossing coins

Suppose you tossed a coin 6 times. What is the probability that you obtained at least 1 tail?

Problem 4 count die practice

Consider that two fair 3-sided die are thrown. As you would expect, the outcome of the two die are independent of each other.

We may express the outcome as a tuple: $(2, 3)$ which indicates that the first die is 2 while the second is 3.

- Let A be the event that the sum of the die is less than or equal to 3.
 - Let B be the event that one die is odd while the other is even.
- i What is the sample space, S , of the experiment?
 - ii What is the probability of each $x \in S$?
 - iii Express all of the elements in the sample space which satisfy A as a set.
 - iv Express all of the elements in the sample space which satisfy B as a set.
 - v Compute $P(A \cap B)$
 - vi Compute $P(A \cap B^c)$

Problem 5 birthday paradox

Assume there are 365 days of the year and people are equally likely to be born on any day. What is the probability that everyone in a room has a unique birthday¹ if there are 10 people in a room? Do not simplify, just write out your expression. (HINT: tree method)

¹For a fun follow-up, consider the following question:

If there are only 4 people in a room we'd expect the probability that two share a birthday to be near 0. However, if there are 300 people in a room we'd expect the probability that two share a birthday to be near 1. How many people are in the room when the probability that two people share a birthday is 50%? Use your answer to the problem above to build an estimate and then see what google has in store for you if you search 'birthday paradox'. Enjoy!