

## HW7: Counting

[HW instructions](#)

### Problem 1 Partner Line

How many ways can we order a line of 168 people where everyone is standing next to their partner? Each person has a unique partner, who is different from themselves, among the 168 people.

One example<sup>1</sup> line is:

57A, 57B, 28B, 28A, 84B, 84A, ...

Where 57A refers to member A of partnership 57.

### Problem 2 Rearrange Digits

How many different numbers can be made by rearranging the digits in the number 7302833? (e.g. 0233378)

### Problem 3 Medley restaurant

- i The restaurant has 52 identical dinner rolls to fill the bread baskets on 16 tables. How many ways can the dinner rolls be distributed to the tables?
- ii A restaurant has 6 ingredients from which to make a salad. How many ways can the restaurant choose exactly 4 ingredients to make a salad?
- iii A restaurant has 3 appetizers, 6 main courses and 4 desserts. How many unique 3-course meals (one appetizer, one main and one dessert) can a diner choose?
- iv After a meal patrons may order coffee, tea, Port Wine, Rum or, nothing at all. How many unique after-meal orders are possible for a table of 8 patrons? Note that the sequencing of drinks ordered is significant<sup>2</sup>.

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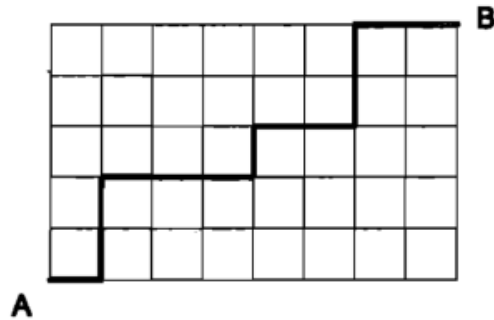
<sup>1</sup>...the beginning of one at least

<sup>2</sup>This restaurant ensures everybody gets their proper drink by counting clockwise around a particular seat at the table. Towards this end (coffee, tea) should be counted distinctly from (tea, coffee), or patrons will get the wrong drinks!

- v 7 tables are all waiting for a check. The waiter only has time to deliver 3 checks in the next minute. How many ways can the waiter order<sup>3</sup> the delivery of 3 checks to any of the 7 tables?

#### Problem 4 Namhattam

The map of a town is depicted below. All of its streets are one-way, so that you can drive only to the right or upwards, as shown above. How many different routes are there to reach point B starting from point A? (One such route is highlighted below).



#### Problem 5 Art Supplies

- i How many ways can one select 6 different colors, from among 15 possible, to put on their palette. Assume that the order of colors on the palette does not matter.
- ii Repeat the problem above, but now assume that the order of colors on the palette yields meaningfully different outcomes.
- iii How many colors can be made by mixing up to 15 colors in equal proportions? For example, if red and blue are among the original 15 colors then mixing them in equal proportions makes purple. Assume that no color could be made by mixing two different subset of the original 15 colors.
- iv A high school has 3 art rooms. The supply closet has 10 tubes of red paint, 7 tubes of blue paint and 5 tubes of green. How many ways can the tubes of paint be distributed among the rooms? (For example:

- room1 gets 4 red, 7 blue and 1 green

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<sup>3</sup>Serving table-3, table-1 and then table-5 should be counted differently than serving table-5, table-1 and then table-3 ... just ask the patrons who really don't like waiting any longer than absolutely necessary!

- room2 gets 3 red, 0 blue and 1 green
- room3 gets 3 red, 0 blue and 3 green

**Problem 6 Challenge cars ★**

A car factory has 74 engines which are compatible with 3 models of cars (model-A, model-B and model-C). How many different deliveries can they make to their 11 dealerships?

For example, the factory might deliver

- 70A, 0B, 0C cars to dealership 1
- 1A, 1B, 1C cars to dealership 2
- 1A, 0B, 0C cars to dealership 3
- no cars to all remaining dealerships because the 74 engines have already been used in the 74 cars above