REC6: Induction, Series

Problem 1 Multiple of 4

Prove by induction that for all n odd positive integers $1 + 3^n$ is divisible by 4.

Problem 2 Fibonacci numbers properties by induction

i. $F_1 - F_2 + F_3 - F_4 + ... + (-1)^n F_{n+1} = (-1)^n F_n + 1$

ii. $F_1 F_2 + F_2 F_3 + F_3 F_4 + ... + F_{2n-1} F_{2n} = F_{2n}^2$

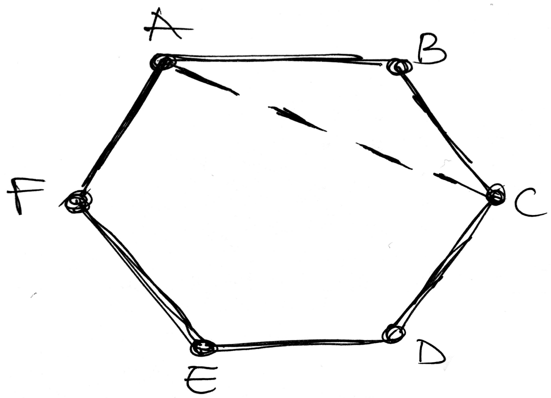
Problem 3 Approximation

i. Let $x > -1$ a real value. Prove by induction over $n \geq 0$ that $(1+x)^n \geq 1 + nx$

ii. Prove that $(\frac{n}{n+1})^n \geq \frac{1}{n+1}$ by using a particular x in the previous inequality.

Problem 4 Polygon sum of angles

Prove that the sum of the interior angles of a convex polygon with n sides is $(n-2)\pi$. You can assume known that the sum of angles of any triangle is π



Problem 5 ★★★ (optional, no credit)

Prove that the inverse-$n \log n$ series diverges : $\sum_{k=2}^{\infty} \frac{1}{n \log n} = \infty$