Summary: NFA and DFA recognize the same languages

We now return to the question:

- Suppose A, B are regular languages, what about
- not A := { w : w is not in A } REGULAR
- A U B := { w : w in A or w in B } REGULAR
- A o B := { w_1 w_2 : w_1 in A and w_2 in B }
- A* := { w_1 w_2 … w_k : $k \geq 0$, w_i in A for every i }

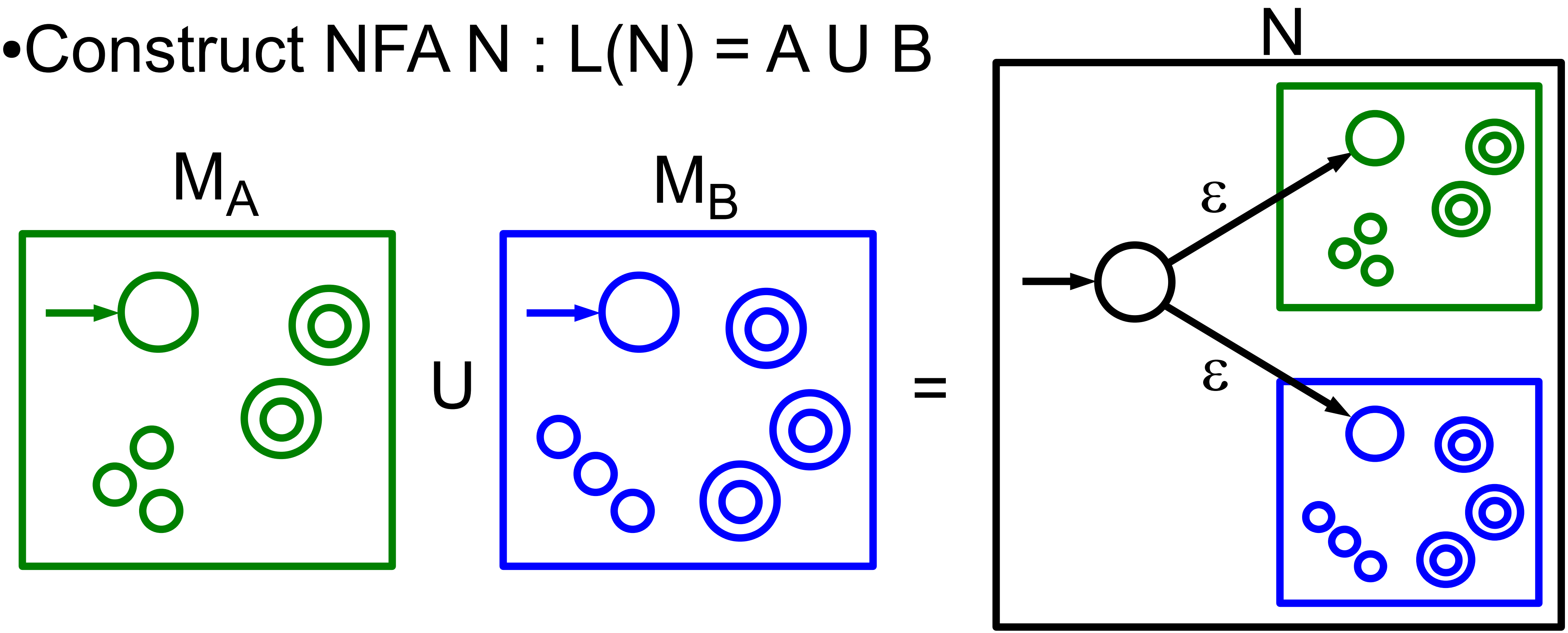
Theorem: If A, B are regular languages, then so is

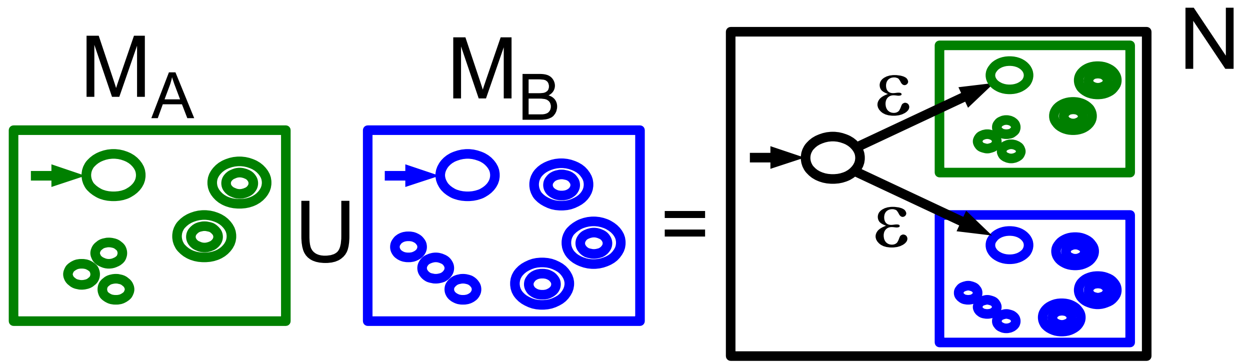
$$A \cup B := \{ w : w \text{ in } A \text{ or } w \text{ in } B \}$$

- Proof idea: Given DFA M_A : $L(M_A) = A$,

  DFA M_B : $L(M_B) = B$,

- Construct NFA N : $L(N) = A \cup B$





Construction:

- Given DFA $M_A = (Q_A, \Sigma, \delta_A, q_A, F_A) : L(M_A) = A$,

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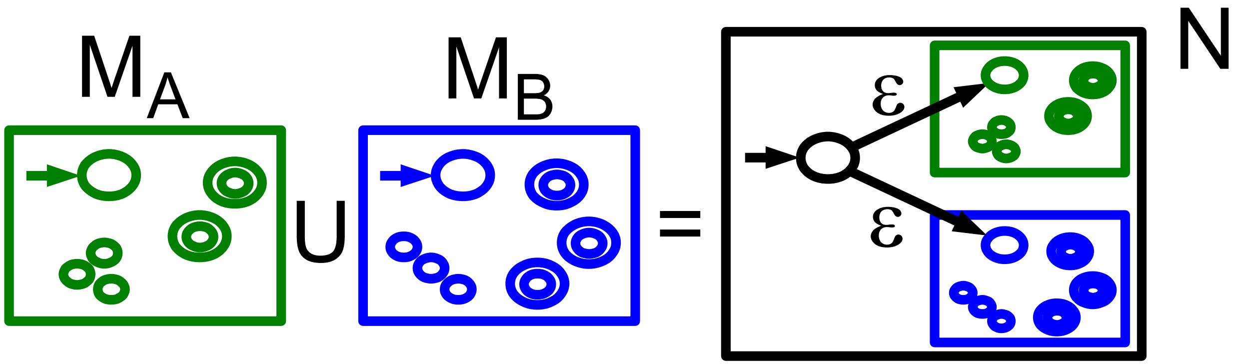
- $Q :=$?



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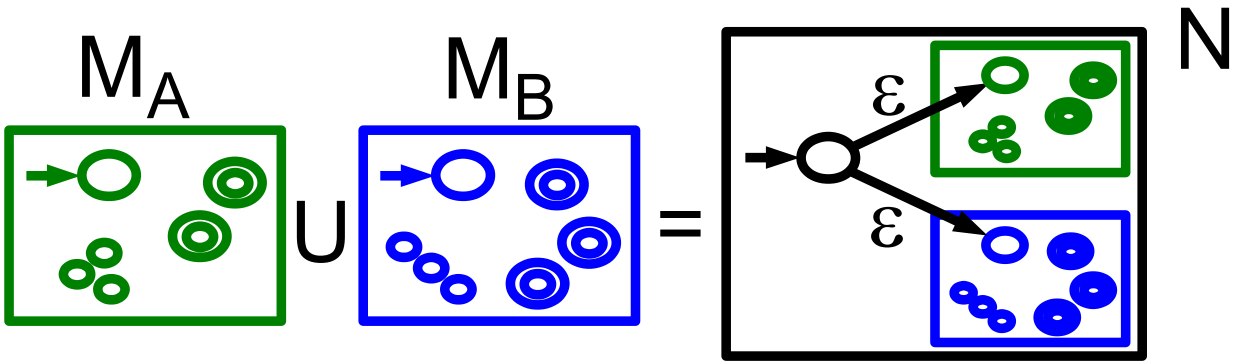

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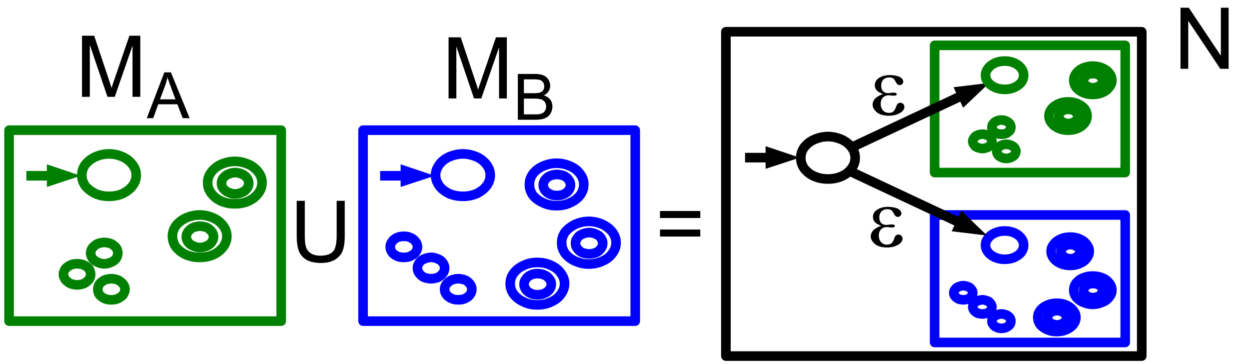

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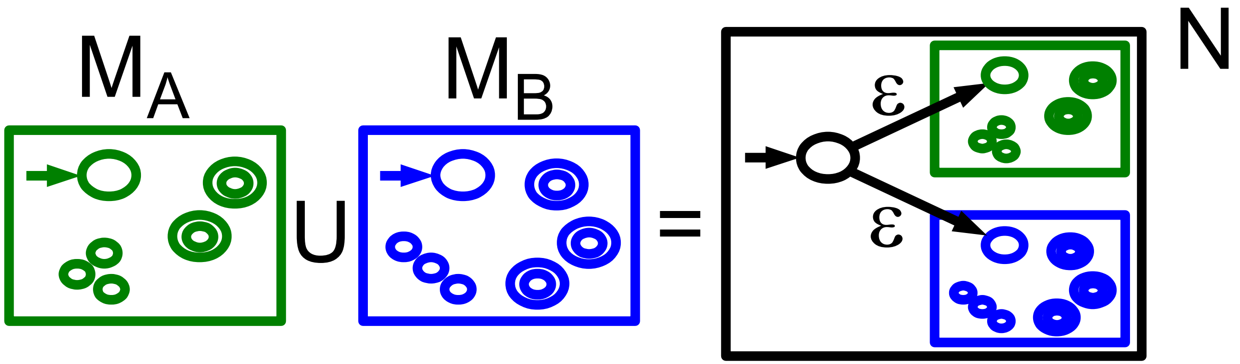

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- $\delta(q,\varepsilon) := \{q_A, q_B\}$
- We have $L(N) = A \cup B$

Example

Is $L = \{w \text{ in } \{0,1\}^* : |w| \text{ is divisible by 3 OR}$
$w \text{ starts with a 1}\}$ regular?

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OR is like U, so try to write $L = L_1 \cup L_2$ where L_1, L_2 are regular

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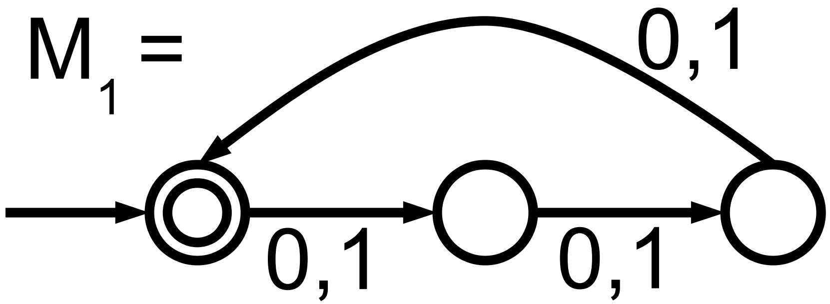
$L_1 = \{w : |w| \text{ is div. by } 3\}$ $L_2 = \{w : w \text{ starts with a } 1\}$

$M_1 =$



$L(M_1) = L_1$

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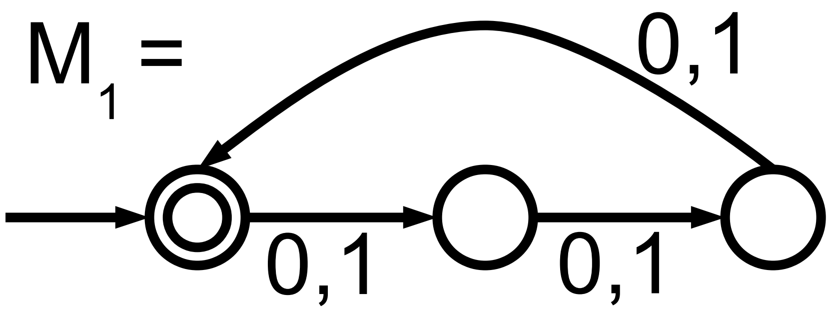
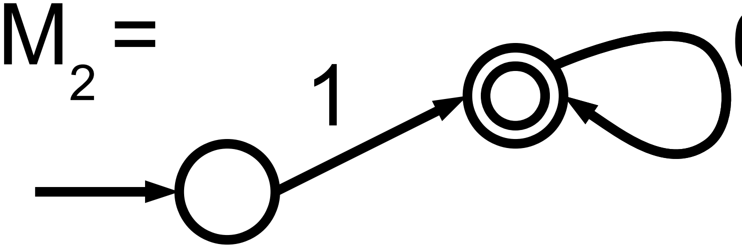


$M_2 =$



$L(M_1) = L_1$ $L(M_2) = L_2$

Example

Is $L = \{w \text{ in } \{0,1\}^* : |w| \text{ is divisible by 3 OR } w \text{ starts with a 1}\}$ regular?

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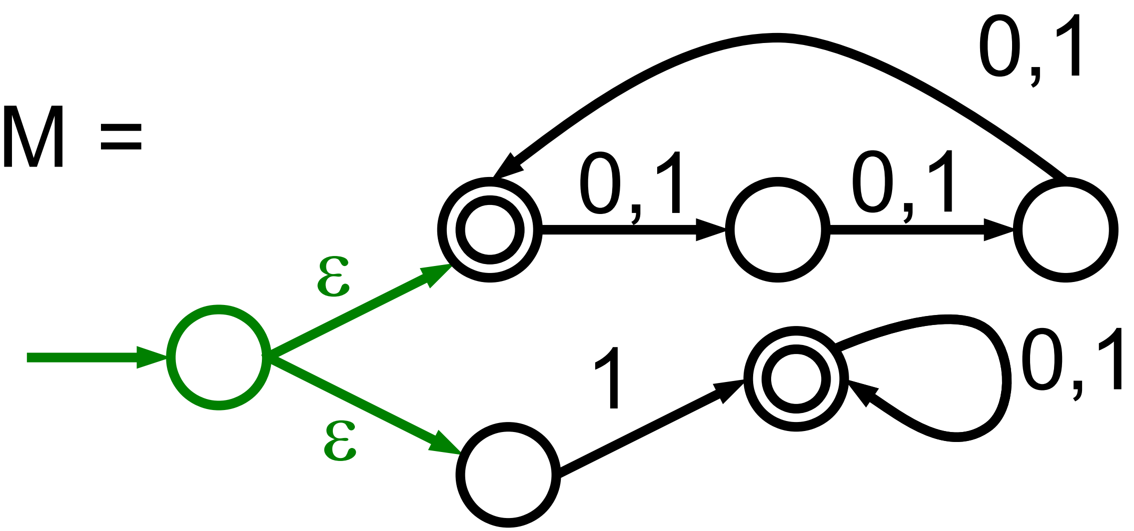

$L(M) = L(M_1) \cup L(M_2)$

$= L_1 \cup L_2$

$= L$

⇒ L is regular.

We now return to the question:

- Suppose A, B are regular languages, then
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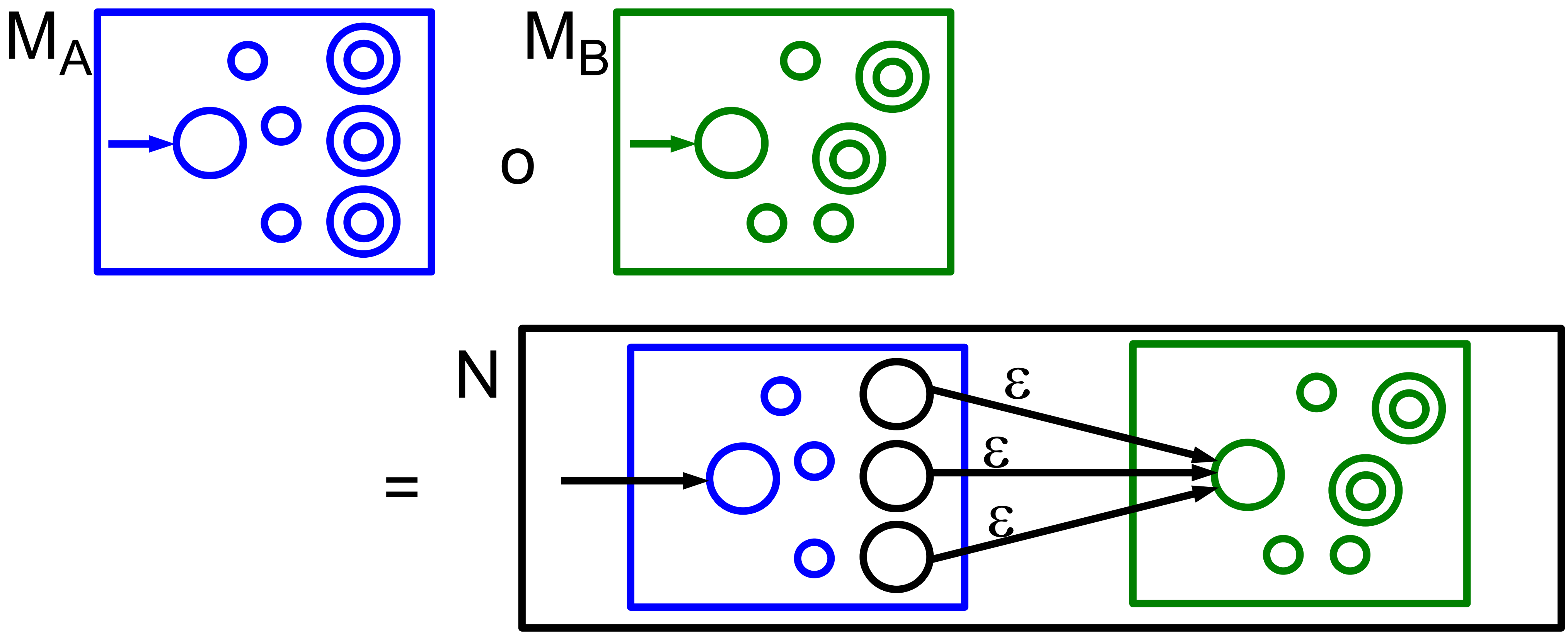
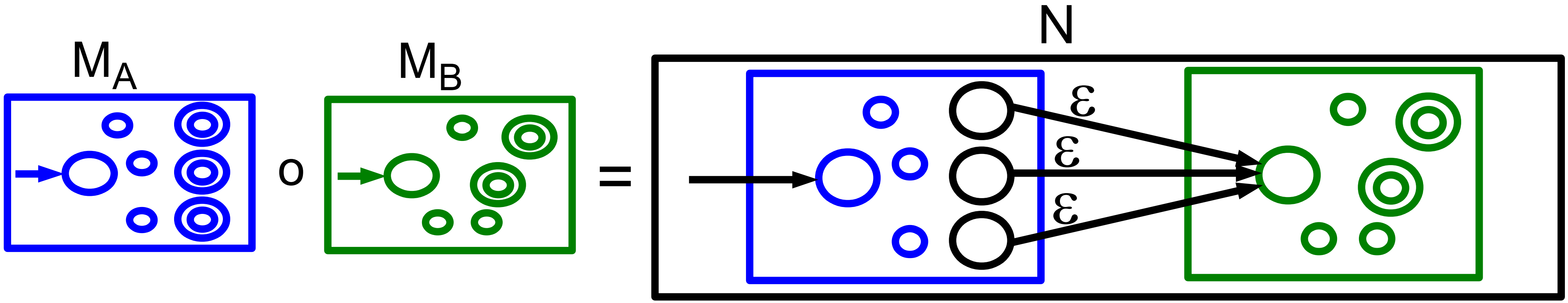
Theorem: If A, B are regular languages, then so is

$A \circ B$:= { w : w = xy for some x in A and y in B }.

- Proof idea: Given DFAs M_A, M_B for A, B construct NFA N : L(N) = $A \circ B$.





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- Given DFA $M_A = (Q_A, \Sigma, \delta_A, q_A, F_A) : L(M_A) = A$,

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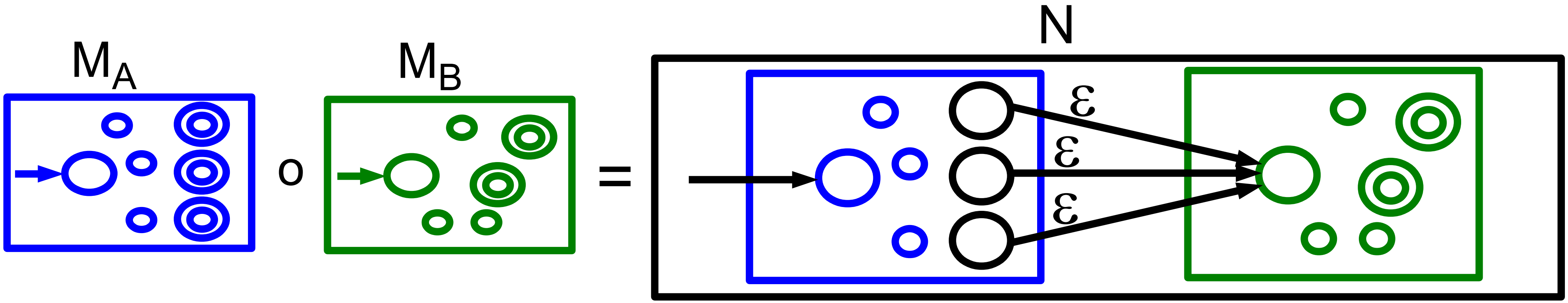

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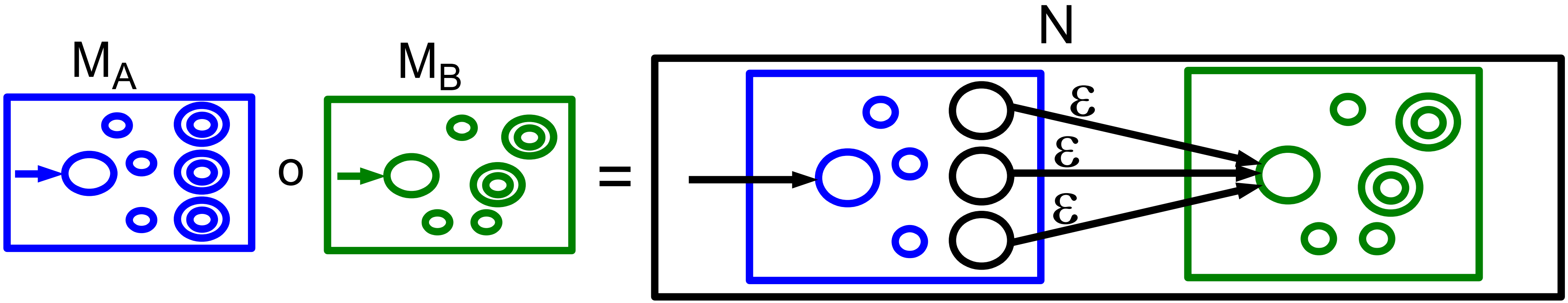

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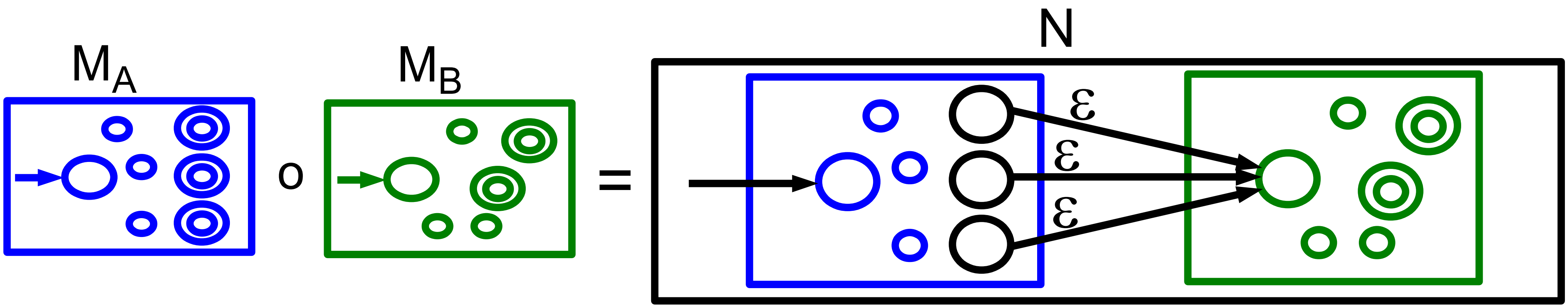

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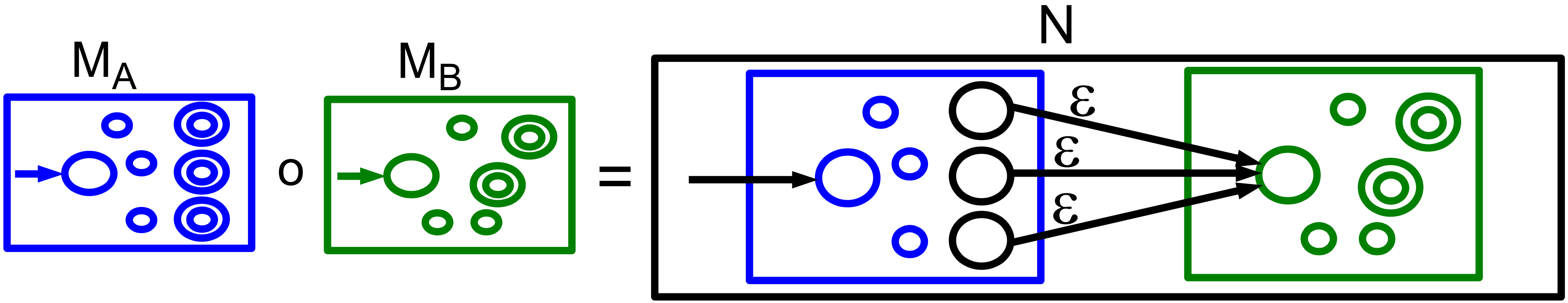

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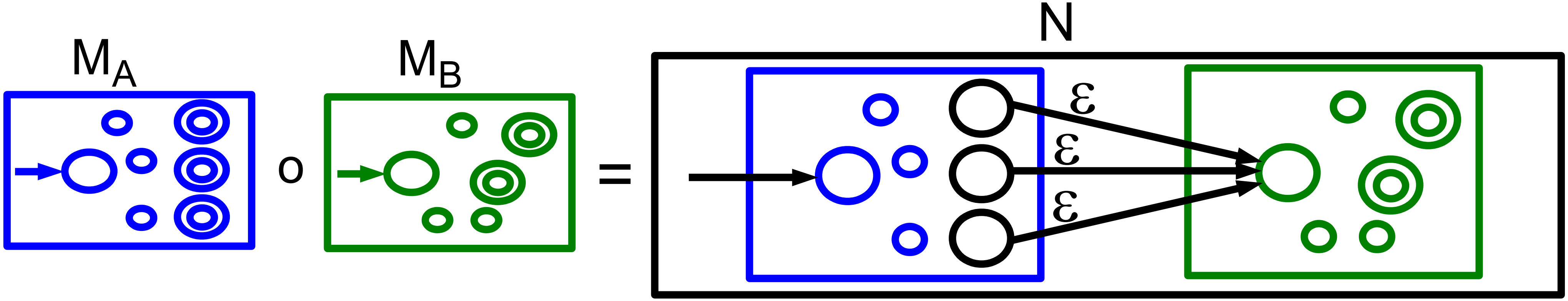

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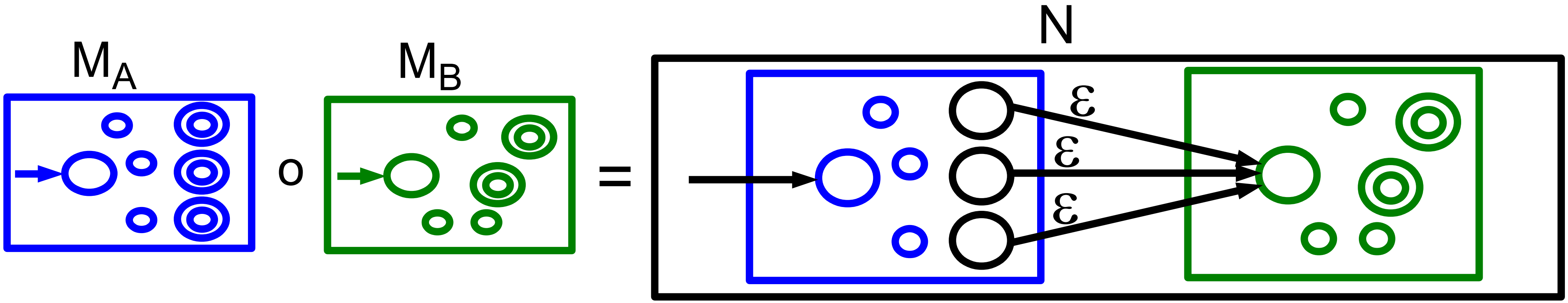

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- We have $L(N) = A \circ B$

Example

Is L = {w in {0,1}* : w contains a 1 after a 0}

regular?

Note: L = {01, 0001001, 111001, … }

Example

Is $L = \{w \text{ in } \{0,1\}^* : w \text{ contains a 1 after a 0}\}$

regular?

Let $L_0 = \{w : w \text{ contains a } 0\}$

$L_1 = \{w : w \text{ contains a } 1\}$. Then $L = L_0 \circ L_1$.

Example

Is L = {w in $\{0,1\}^*$: w contains a 1 after a 0}

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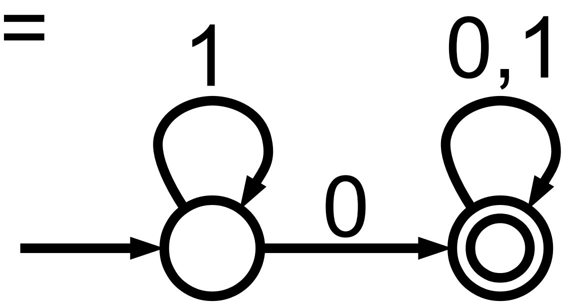
M_0 =



$L(M_0) = L_0$

Example

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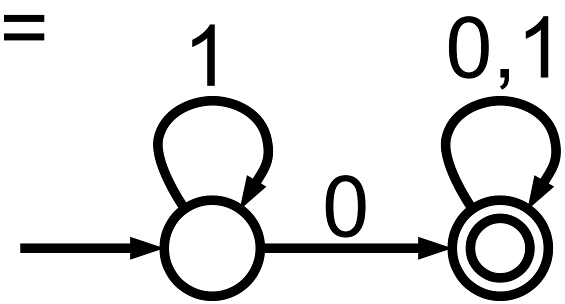
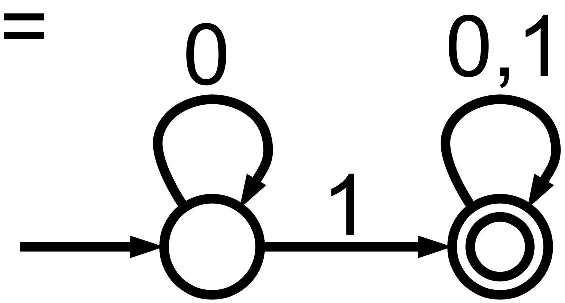


$L(M_0) = L_0$

$M_1 =$



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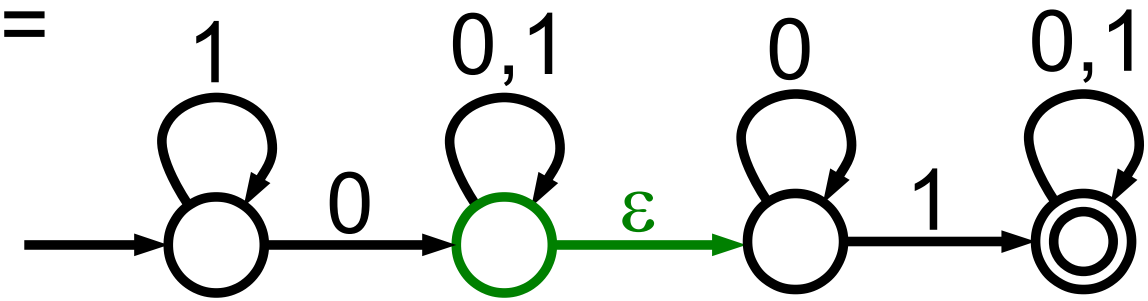
M =



$L(M) = L(M_0) \circ L(M_1) = L_0 \circ L_1 = L$

$\Rightarrow$ L is regular.

We now return to the question:

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Theorem: If A is a regular language, then so is

$A^* := \{ w : w = w_1 \ldots w_k, \; w_i \text{ in } A \text{ for } i=1,\ldots,k \}$

- Proof idea: Given DFA M_A : $L(M_A) = A$,

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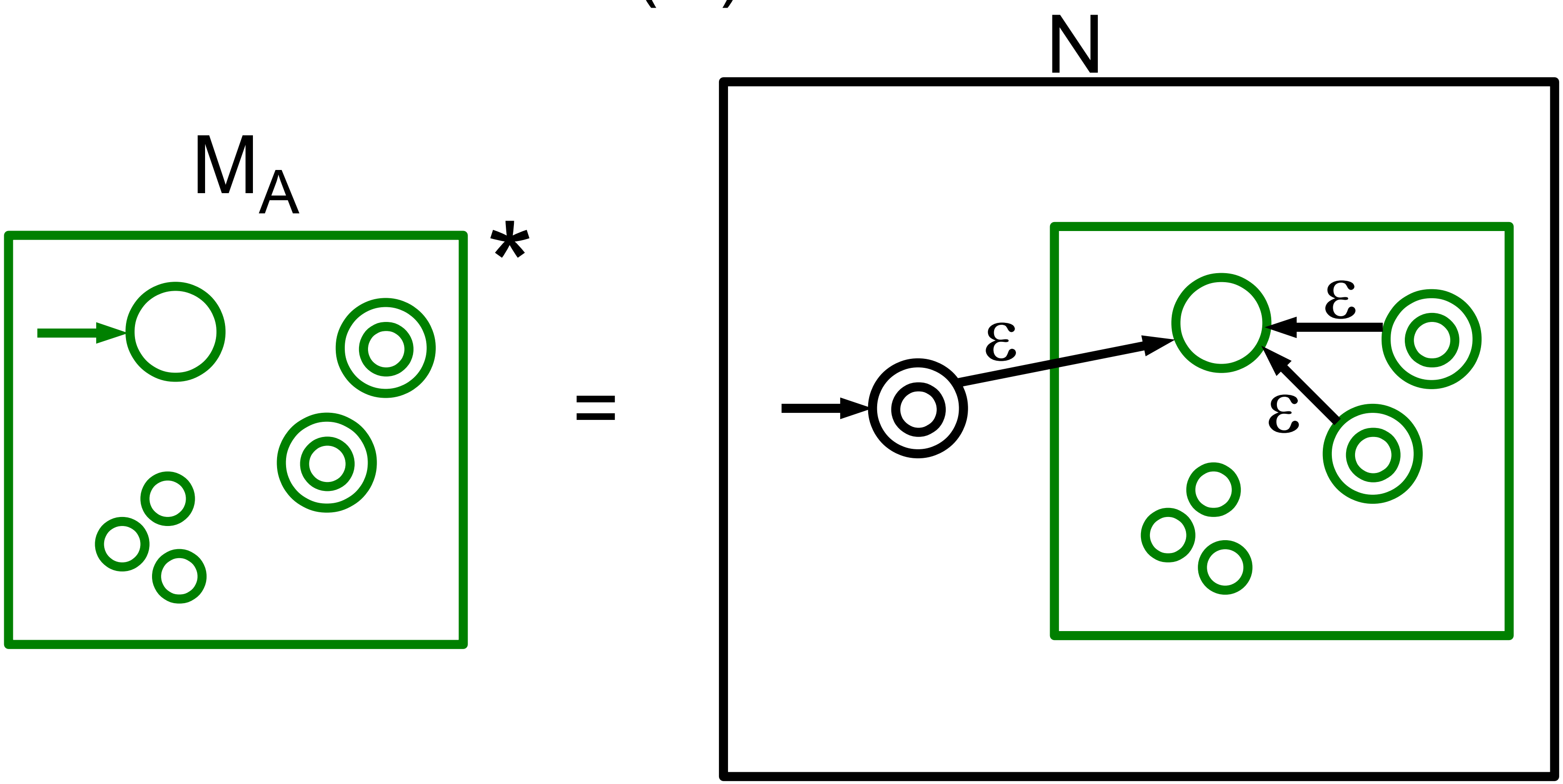

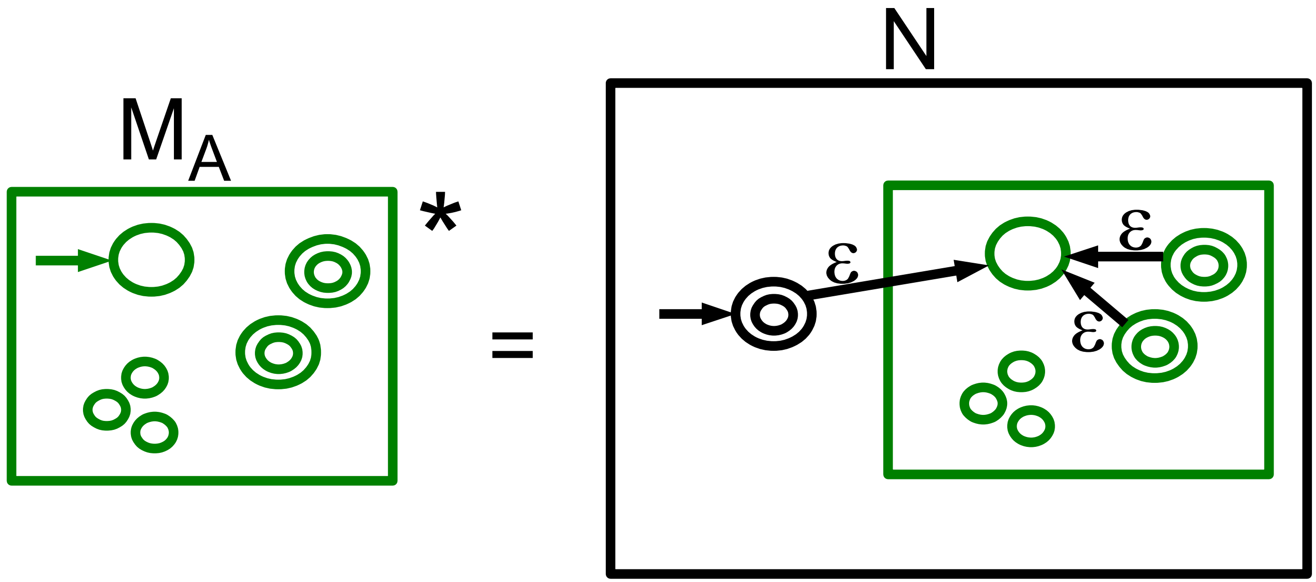

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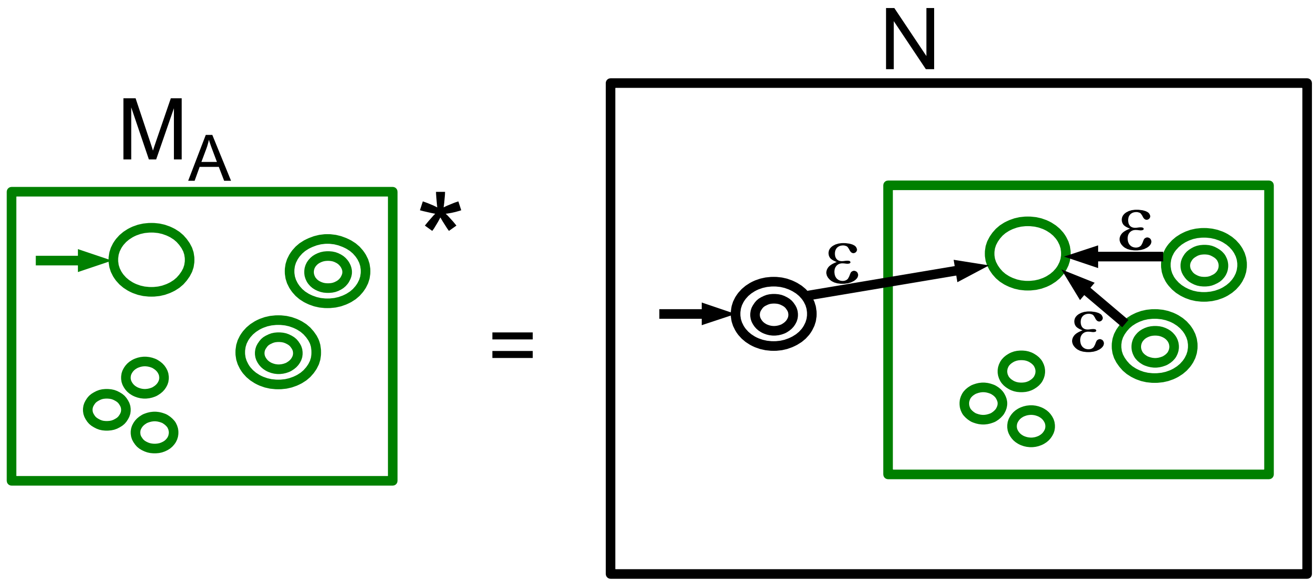

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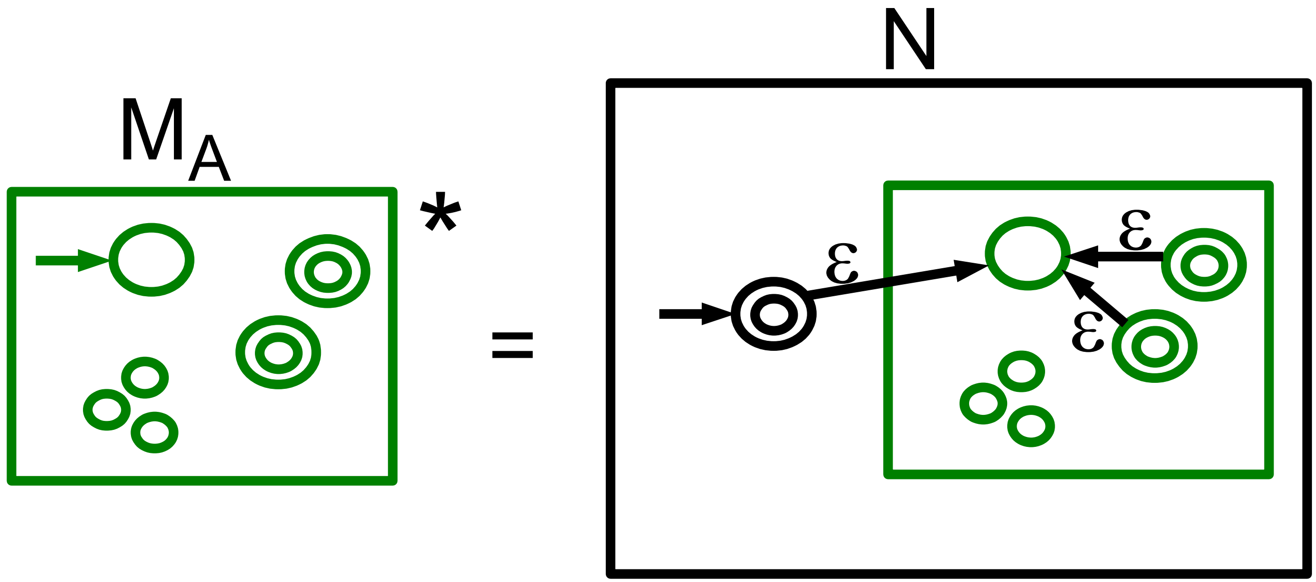

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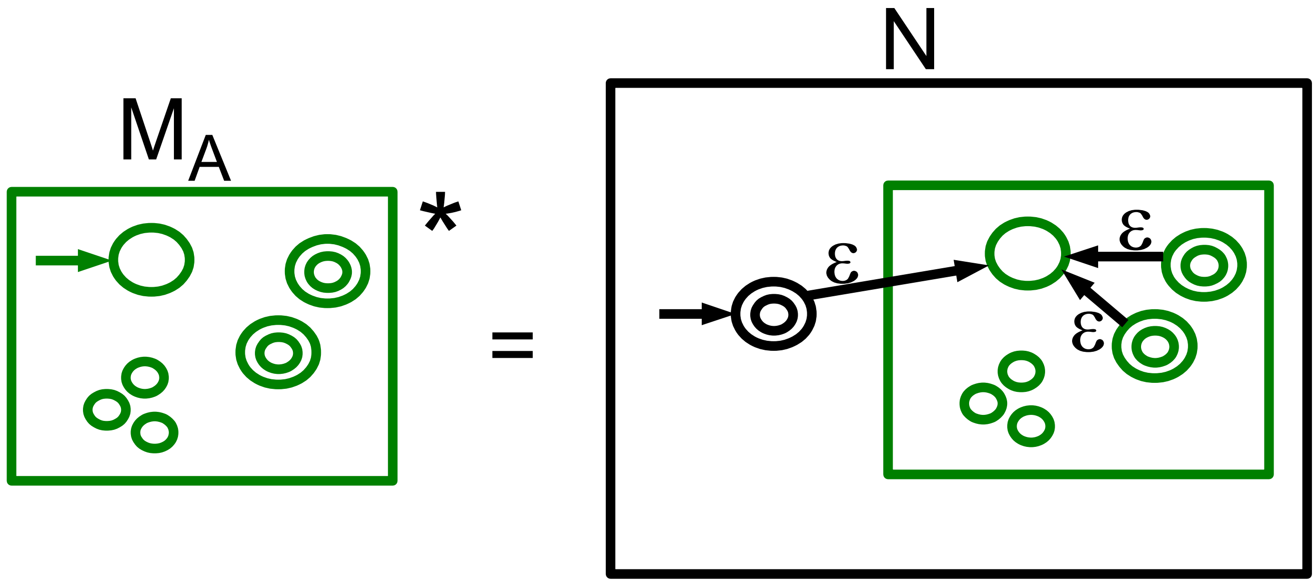

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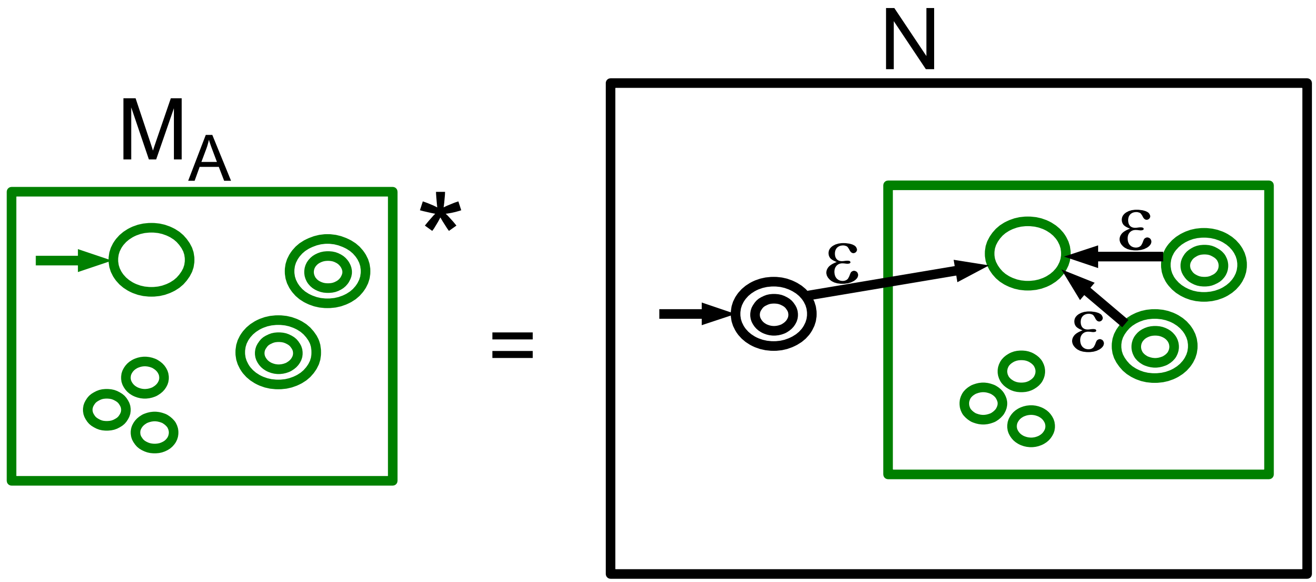

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Is L = {w in {0,1}* : w has even length}

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Let L_0 = {w : w has length = 2}. Then L = L_0*.

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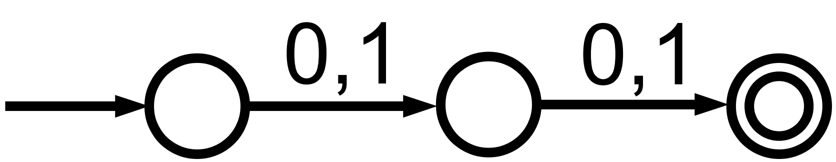
$M_0 =$



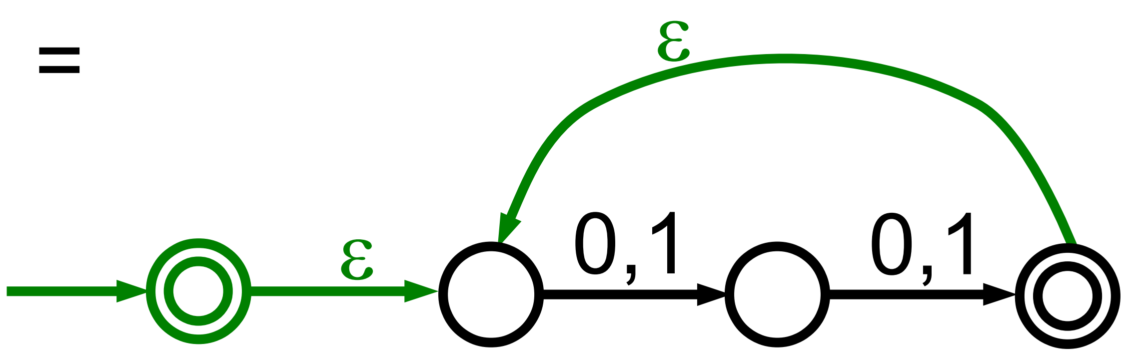
$L(M_0) = L_0$

Example

Is $L = \{w \text{ in } \{0,1\}^* : w \text{ has even length}\}$ regular?

Let $L_0 = \{w : w \text{ has length} = 2\}$. Then $L = L_0^*$.

M =



$$L(M) = L(M_0)^* = L_0^* = L$$

⇒ L is regular.

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are all regular!

We now return to the question:

- Suppose A, B are regular languages, then
- not A := { w : w is not in A }
- A U B := { w : w in A or w in B }
- A o B := { w_1 w_2 : w_1 in A and w_2 in B }
- A* := { w_1 w_2 … w_k : $k \geq 0$, w_i in A for every i }

What about $A \cap B$:= { w : w in A and w in B } ?

We now return to the question:

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De Morgan's laws: A ∩ B = not ((not A) U (not B))

By above, (not A) is regular, (not B) is regular,

(not A) U (not B) is regular,

not ((not A) U (not B)) = A ∩ B regular

We now return to the question:

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are all regular