

Quiz 5

Name: _____

(3 + 3.5 + 3.5 = 10 points) Crucial edges in a network flow

Consider a network flow (directed) graph G with a positive integer capacity on each edge, a source s , and a sink t . We say that an edge e is *crucial* if decreasing its capacity by one will decrease the maximum flow from s to t by one.

- (a) Give an example network flow graph in which at least two edges are crucial and at least three edges are not crucial. Mark each edge as crucial or not crucial.

Recall that an edge e is a *bottleneck* if increasing its capacity by one will increase the maximum flow from s to t by one. In the following two parts, indicate whether the given statement is true or false. If you claim that the statement is true, give a brief proof; otherwise, give a counterexample.

- (b) True or False: If an edge is crucial, then it is also a bottleneck.

(c) True or False: If an edge is a bottleneck, then it is also crucial.