(Optional) Problem Set 6

1. (6 points) Variants of IP

Let IP_1 be the class obtained if we change the soundness error of IP from 1/3 to 0, leaving the completeness error unchanged. Let IP_2 be the class obtained if we change the completeness error of IP changed from 1/3 to 0, leaving the soundness error unchanged. What are the classes IP_1 and IP_2; i.e., how are they related to IP and other classes we have studied? Justify your answers.

2. (3 × 6 = 18 points) For each of these statements, indicate whether it is true, it is false, or we do not know. In each case, justify your answer.

(a) If R is a regular language and N is a non-regular language, then $N - R$ is always non-regular.

(b) If L_1 and L_2 are in NL, then $L_1 - L_2$ is in NL.

(c) For any two languages L_1 and L_2 in NP, if $L_1 \leq_P L_2$, then $L_2 \leq_P L_1$.

3. (6 points) Maximum independent set

An independent set of a graph $G = (V, E)$ is a subset S of V such that no two vertices in S have an edge between them in E. It is well known that the following language is NP-complete.

$$\{\langle G, k\rangle : G \text{ has an independent set of size } k\}$$

Show that if P = NP, then a maximum independent set of any graph can be found in polynomial time.