CSU390 Theory of Computation	Assignment #05
Spring 2006	February 24, 2006

Homework 05

Due: Friday, March 3, 2006

Instructions

1. Please review the homework grading policy outlined in the course information page.
2. On the *first page* of your solution write-up, you *must* make explicit which problems are to be graded for regular credit, which problems are to be graded for extra credit, and which problems you did not attempt. Use a table that looks like this:

Problem	1	2	3	4	5	6	7	8	9	...
Credit	RC	RC	RC	EC	RC	EC	NA	NA	EC	...

   where "RC" denotes "regular credit", "EC" denotes "extra credit", and "NA" denotes "not attempted". Failure to include such a table will result in an arbitrary set of problems being graded for regular credit, no problems being graded for extra credit, and a 5% penalty assessment.
3. You must also write down with whom you worked on the assignment. If this varies from problem to problem, write down this information separately with each problem.

Problems

Required: 4 of the following 6 problems
Points: 25 points per problem

1. Give both an informal description and a state transition diagram for a PDA that recognizes the language

$$\left\{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k \mid i,j,k \geq 0 \text{ and } i = j \text{ or } j = k\right\}$$

2. Give both an informal description and a state transition diagram for a PDA that recognizes the language

$$\left\{x\#y \mid x, y \in \{0,1\}^+ \text{ and } |x| \leq |y| \text{ and the } n^{\text{th}} \text{ symbol of } x \text{ matches the } n^{\text{th}} \text{ symbol of } y, \text{ where } n = |x|\right\}$$

3. Do Problem 2.30(a).
4. Do Problem 2.31.
5. Consider the following language:

$$L = \left\{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k \mid i,j,k \geq 0 \text{ and } k = \min(i,j)\right\}$$

   Assume that this language is context-free, and let p be a pumping length for it. Let $s = \mathtt{a}^p\mathtt{b}^p\mathtt{c}^p$.

   (a) Show that $s \in L$ and that $|s| \geq p$.

   (b) Show that s can be split into five pieces, $s = uvxyz$, such that conditions 2 and 3 of the Pumping Lemma for context-free languages are satisfied and $uv^ixy^iz \in L$ for all $i \geq 1$ (so s can be pumped up any number of times).

(c) Show that s can also be split into five pieces, $s = u'v'x'y'z'$, such that conditions 2 and 3 of the Pumping Lemma are satisfied and $u'x'z' \in L$ (so s can also be pumped down).

(d) Prove that, nevertheless, L violates the Pumping Lemma for context-free languages, so it is not context-free.

6. Prove that if a unary language fails to satisfy the Pumping Lemma for regular languages then it must also fail to satisfy the Pumping Lemma for context-free languages. *Hint:* Concatenation of strings over a unary alphabet is commutative.