Homework 04

Due: Friday, February 24, 2006

Instructions

1. Please review the homework grading policy outlined in the course information page.
2. On the *first page* of your solution write-up, you *must* make explicit which problems are to be graded for regular credit, which problems are to be graded for extra credit, and which problems you did not attempt. Use a table that looks like this:

Problem	1	2	3	4	5	6	7	8	9	...
Credit	RC	RC	RC	EC	RC	EC	NA	NA	EC	...

   where "RC" denotes "regular credit", "EC" denotes "extra credit", and "NA" denotes "not attempted". Failure to include such a table will result in an arbitrary set of problems being graded for regular credit, no problems being graded for extra credit, and a 5% penalty assessment.

3. You must also write down with whom you worked on the assignment. If this varies from problem to problem, write down this information separately with each problem.

Problems

Required: 5 of the following 7 problems
Points: 20 points per problem

1. Do the following:
   - Draw the two parse trees corrresponding to the two different leftmost derivations given in the solution to Exercise 2.8.
   - Do Problem 2.27(a). *Hint:* The ambiguity present in this grammar is due to the `if-then` and `if-then-else` statements. The ambiguity you will discover is referred to as the "dangling else" ambiguity.
2. Do Exercise 2.4 (b,c,e). For each grammar, as in the examples done in class, provide annotation indicating what each variable generates.
3. - Construct a context-free grammar for the following language:

$$\{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k \mid i,j,k \geq 0 \text{ and } j = i+k\}$$

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   In both cases, for full credit you must supply a brief description of how you arrived at your solution and provide annotation indicating what each variable generates, as in the examples done in class.

4. Construct a context-free grammar for the following language:

$$\left\{ \mathtt{a}^i \mathtt{b}^j \mathtt{c}^k \mid i, j, k \geq 0 \text{ and } i \neq j \text{ or } j \neq k \right\}$$

For full credit you must supply a clear description of how you arrived at your solution and provide annotation indicating what each variable generates, as in the examples done in class.

5. Give a state transition diagram for a PDA that recognizes the language described in Exercise 2.6(c). *Hint:* Implement the informal description given in the solution to Exercise 2.7(c).

6. Prove that if L is a context-free language then $L^{\mathcal{R}}$, the language obtained by reversing all the strings in L, is context-free.

7. Given a string w, if $w = xyz$, we say x is a *prefix* of w, z is a *suffix* of w, and y is a *substring* of w. (The strings x, y, and z are allowed to be arbitrary strings, including the empty string, so clearly any prefix or any suffix of w is also a substring of w, and each of ε and w is also a prefix, suffix, and substring of w.) For any language L, define

   - Prefixes$(L) = \{x \mid x$ is a prefix of some $w \in L\}$,
   - Suffixes$(L) = \{z \mid z$ is a suffix of some $w \in L\}$, and
   - Substrings$(L) = \{y \mid y$ is a substring of some $w \in L\}$.

   Prove that if L is context-free, then:

   a. Prefixes(L) is context-free.
   b. Suffixes(L) is context-free.
   c. Substrings(L) is context-free.

   You may use the result of Problem 6.