

AdaBoost

Given: $(x_1, y_1), \dots, (x_m, y_m)$ $x_i \in \mathcal{X}, y_i \in \{-1, +1\}$
Initialize: $D_1(i) = 1/m$ (uniform distribution)

For $t = 1, \dots, T$

1. train weak learner using distribution D_t , obtaining

$$h_t : \mathcal{X} \rightarrow \{-1, +1\}$$

2. assess error rate of h_t w.r.t. D_t ,

$$\epsilon_t = \Pr_{D_t}[h_t(x) \neq y]$$

3. assign confidence to h_t ,

$$\alpha_t = \frac{1}{2} \ln \frac{1 - \epsilon_t}{\epsilon_t}$$

4. update distribution, increasing weight were incorrect and vice versa

$$D_{t+1}(i) = \frac{D_t(i)e^{-\alpha_t y_i h_t(x_i)}}{Z_t}$$

where $Z_t = \sum_i D_t(i)e^{-\alpha_t y_i h_t(x_i)}$ is a normalization factor. Note that

$$e^{-\alpha_t y_i h_t(x_i)} = \begin{cases} \sqrt{\frac{\epsilon_t}{1-\epsilon_t}} & h_t(x_i) = y_i \\ \sqrt{\frac{1-\epsilon_t}{\epsilon_t}} & h_t(x_i) \neq y_i \end{cases}$$

Output final hypothesis:

$$f(x) = \sum_{t=1}^T \alpha_t h_t(x)$$

$$\begin{aligned} H(x) &= \text{sign}(f(x)) \\ &= \text{sign}\left(\sum_{t=1}^T \alpha_t h_t(x)\right) \end{aligned}$$