SETS

A set is a collection of distinct elements

$B = \{\text{France}, \text{Portugal}, \text{Andorra}\}$ = countries bordering mainland Spain.

oops... Gibraltar?

$S = \{\text{basketball}, \text{soccer}\}$ = sports that I like.

Natural numbers : $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ → Some sources: no zero.

Integers : $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

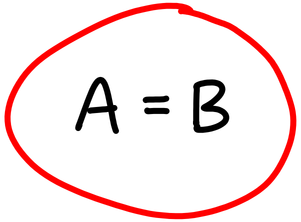
Positive integers : $\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$

Rational numbers: $\mathbb{Q}$ e.g., $\frac{1}{3}, \frac{5}{7}, \frac{13}{2}, 8$ integer divided by integer

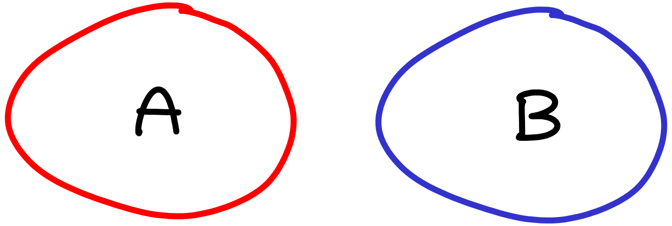
Irrational numbers e.g., $\pi, e, \sqrt{2}$

Real numbers : $\mathbb{R}$ all rational & irrational

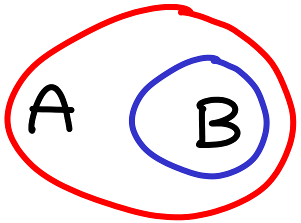
Venn diagrams



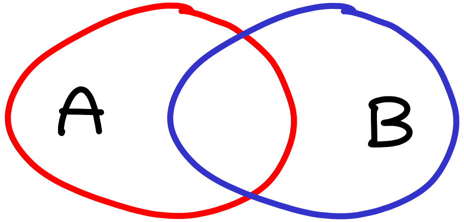
equal sets



disjoint sets



B subset of A



general picture

E : even natural numbers = $\{0, 2, 4, \ldots\}$

E is a subset of $\mathbb{N}$: $E \subseteq \mathbb{N}$

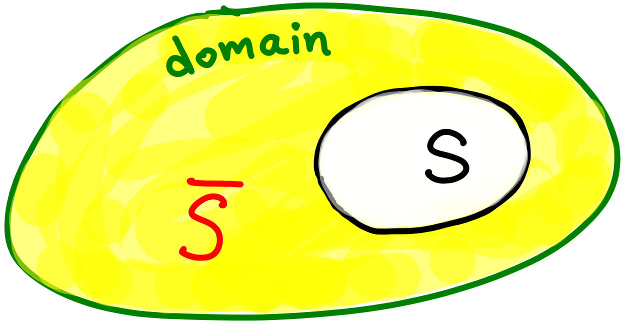
In fact it is a strict subset: $E \subset \mathbb{N}$

Useful property:

If S has n elements,
number of subsets of $S = 2^n$.

(all combinations of
including or not including
each element)

Complement of a set $S = \overline{S}$ (contains all elements not in S)



This requires context: a known domain.

Essentially S is a subset of the domain.

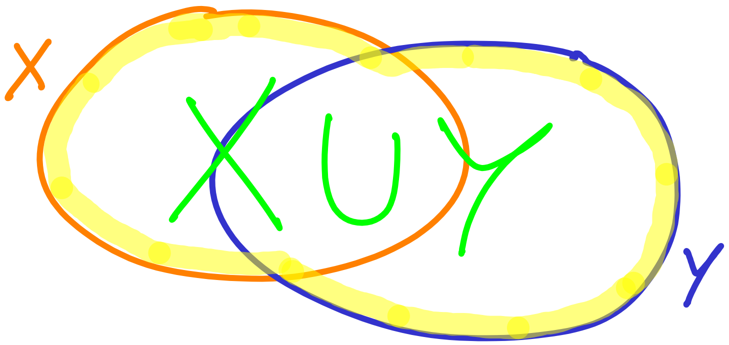
e.g., domain = $\mathbb{Z}^+$ S = powers of 2, $\overline{S} = \{3, 5, 6, 7, 9, \ldots\}$

$X = \{1, 3, 5\}$ $\quad$ $Y = \{8, 5, 7\}$

Union of sets: include every element present.

$X \cup Y = \{1, 8, 3, 5, 7\}$

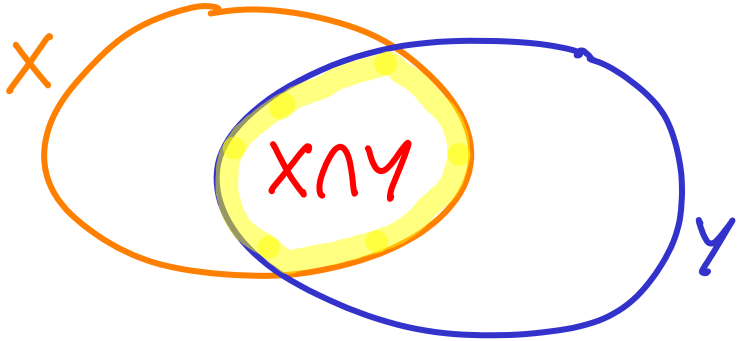
$p \in X \cup Y$ iff $p \in X$ OR $p \in Y$



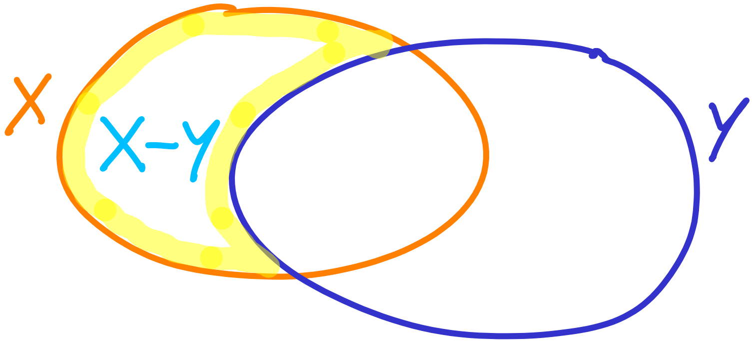
Intersection of sets: include common elements.

$X \cap Y = \{5\}$

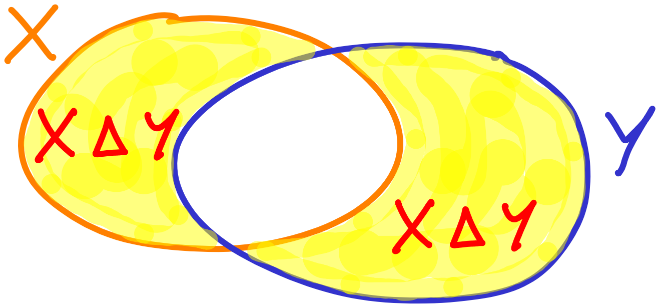
$p \in X \cap Y$ iff $p \in X$ AND $p \in Y$



Set difference: $X - Y$ $\qquad$ $p \in X - Y$ iff $p \in X$ AND $p \notin Y$

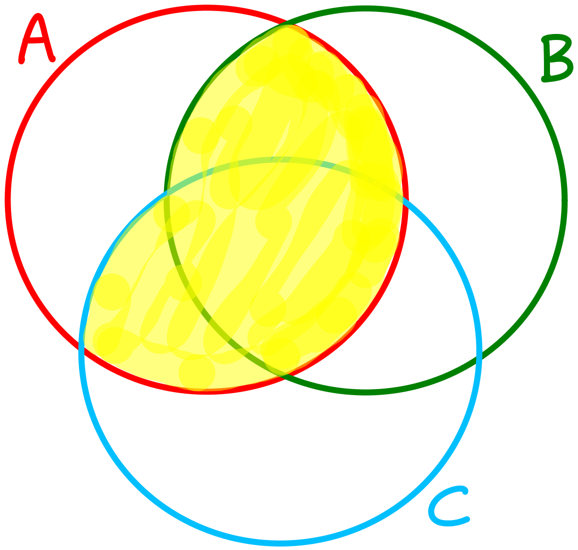


Symmetric difference: $X \Delta Y$ $\qquad$ $= (X-Y) \cup (Y-X)$



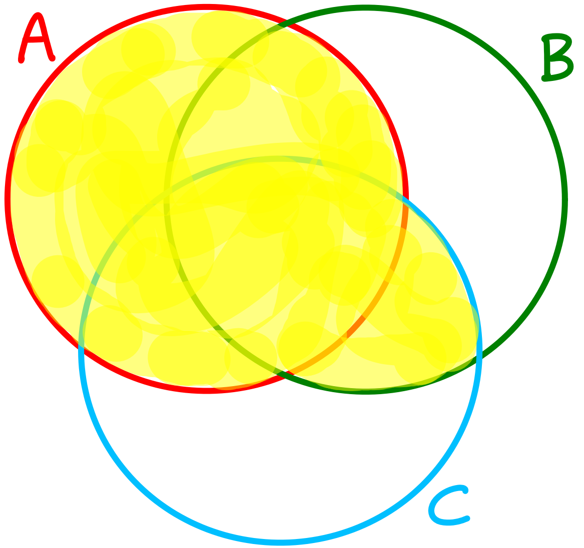
Distributive Laws for sets:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



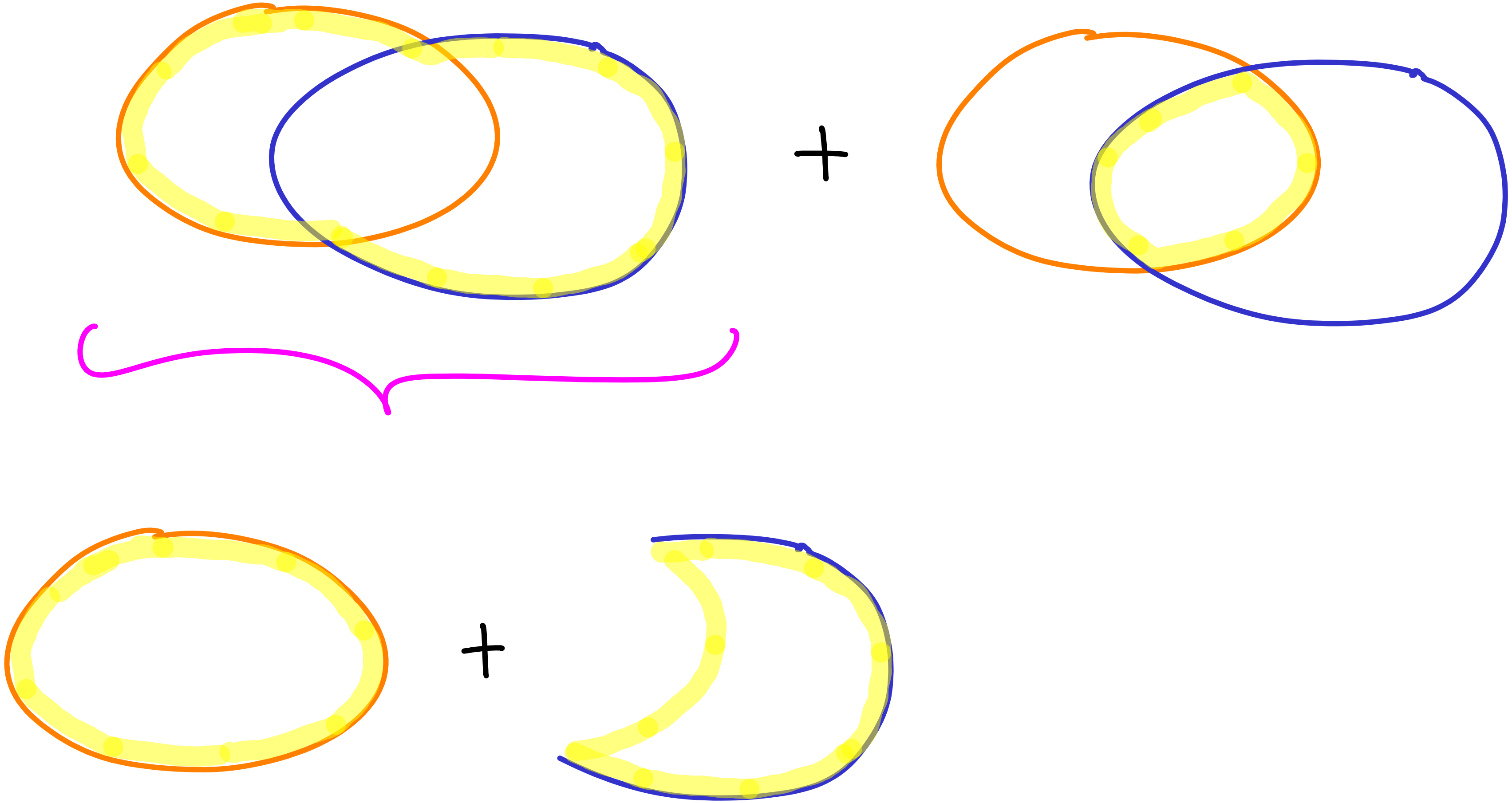
Which seems like a simpler description?

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$



Which seems like a simpler description?

$$|A| + |B| = |A \cup B| + |A \cap B|$$



see **Inclusion-Exclusion Principle**

Set builder notation: use when description of set isn't "basic"

$\{5, 14, 19, 23, 28, 32, 37, 41, 46, 50, 55, 64, 69, 73, 78 \ldots\}$

$S = \{x \in \mathbb{N} \mid \text{sum of digits in } x \text{ is a multiple of } 5\}$

implied: "all x"

"such that" or "for which"

How many integers in $\{1, \ldots, 1000\}$ are divisible by 2 or 5?

$$A = \{x \in \mathbb{Z} \mid 1 \leq x \leq 1000 \text{ and } 2|x\}$$

$$|A \cup B| = ?$$

$$B = \{x \in \mathbb{Z} \mid 1 \leq x \leq 1000 \text{ and } 5|x\}$$

$$|A| = 500$$

$$x \in A \cap B \text{ iff } 10|x$$

$$|B| = 200$$

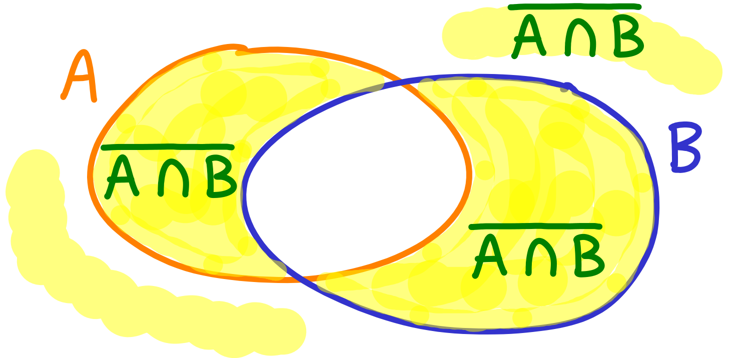
$$|A \cap B| = 100$$

$$|A| + |B| = |A \cup B| + |A \cap B| \longrightarrow |A \cup B| = 600$$

De Morgan's Laws

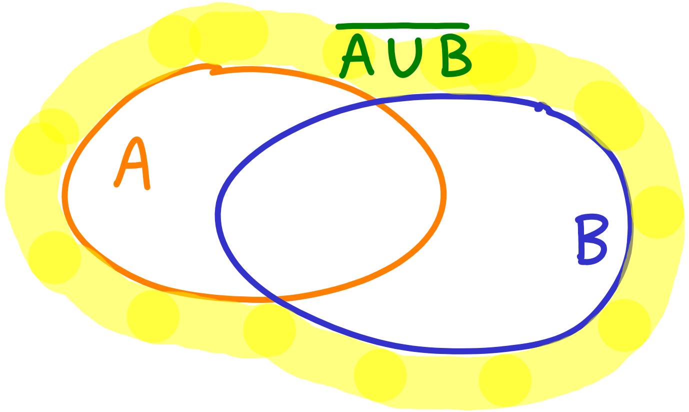
for propositions: $\neg(A \text{ AND } B) \leftrightarrow \neg A \text{ OR } \neg B$

for sets: $\overline{A \cap B} = \overline{A} \cup \overline{B}$



for propositions: $\neg(A \text{ OR } B) \leftrightarrow \neg A \text{ AND } \neg B$

for sets: $\overline{A \cup B} = \overline{A} \cap \overline{B}$



SEQUENCES

Like sets, they are a collection of elements.

2 main differences:

- repeats OK $(a, b, a, b, a, a, b, b, \ldots)$
- order matters $(a, b, c) \neq (c, b, a)$

Cartesian product of sets

produces a set of sequences.

each sequence:

one element per set.

$B = \{\text{egg}, \text{banana}\}$

$L = \{\text{soup}, \text{salad}\}$

$D = \{\text{egg}, \text{steak}, \text{cake}\}$

$$B \times L \times D = \{(\text{egg}, \text{soup}, \text{egg}), (\text{egg}, \text{soup}, \text{steak}), (\text{egg}, \text{soup}, \text{cake}),$$
$$(\text{egg}, \text{salad}, \text{egg}), (\text{egg}, \text{salad}, \text{steak}), (\text{egg}, \text{salad}, \text{cake}),$$
$$(\text{banana}, \text{soup}, \text{egg}), (\text{banana}, \text{soup}, \text{steak}), (\text{banana}, \text{soup}, \text{cake}),$$
$$(\text{banana}, \text{salad}, \text{egg}), (\text{banana}, \text{salad}, \text{steak}), (\text{banana}, \text{salad}, \text{cake})\}$$

Cartesian product of sets

produces a set of sequences

If we take the product of n copies of S, we write S^n

e.g., $\{0,1\}^3 = \{(0,0,0),(0,0,1),(0,1,0),(1,0,0),(0,1,1),(1,0,1),(1,1,0),(1,1,1)\}$

set of corners of a cube

Similar to the "useful property" mentioned earlier, about number of subsets.