

# CY 2550 Foundations of Cybersecurity

Cryptography Part 3

January 27

Alina Oprea

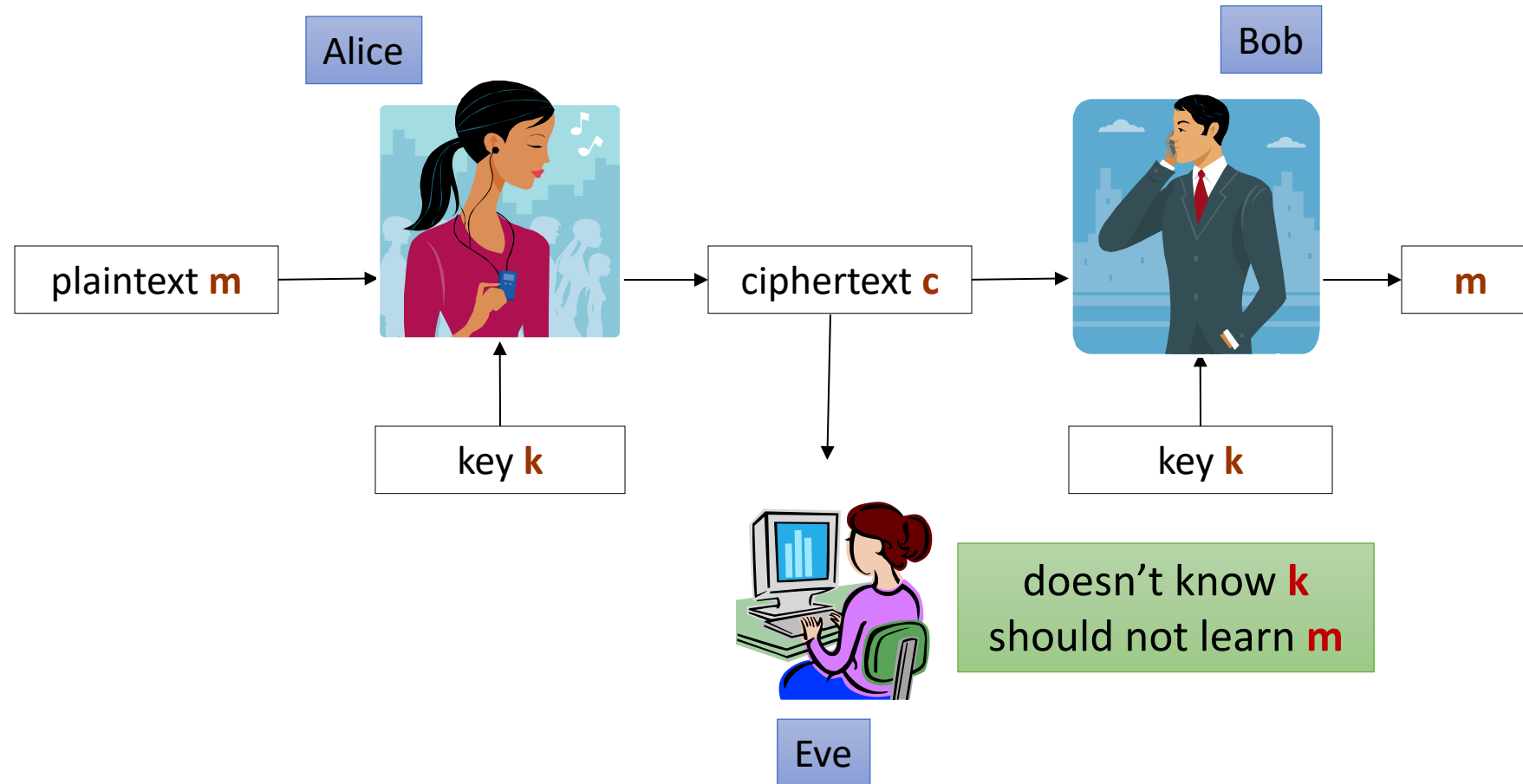
Associate Professor, Khoury College

Northeastern University

# Outline

- Symmetric-key crypto
- Definitions of security for encryption
- Block ciphers and modes of operation
- Public-key crypto
- Key exchange

# Encryption Terminology



Encryption scheme = encryption & decryption procedures

# One-time pad is perfectly secure!

$\ell$  – a parameter  
 $\mathcal{K} = \mathcal{M} = \{0,1\}^\ell$

component-wise **xor**

Vernam's cipher:

$$\text{Enc}_k(m) = k \oplus m$$

$$\text{Dec}_k(c) = k \oplus c$$



Gilbert  
Vernam  
(1890 –1960)

Correctness:

$$\text{Dec}_k(\text{Enc}_k(m)) = k \oplus (k \oplus m) \\ m$$

# Computational security

**Restriction:**

**Eve is computationally-bounded**

We will construct schemes that in **principle can be broken** if the adversary has a **huge computing power** or is **extremely lucky**.

- E.g., break the scheme by **enumerating** all possible secret keys. ( **“brute force attack”** )
- E.g., break the scheme by **guessing** the secret key.

**Goal:** cannot be broken with **reasonable computing** power with **reasonable probability**.

# Eavesdropping security

- Ciphertext INDistinguishability under an EAVesdropping attacker (IND-EAV)

Charlie (Challenger)



$k, Enc_k$

Adv



$m_0, m_1 \in \mathcal{M}$

Round 1: Charlie chooses  $k$  and encryption algo

Round 2: Adv chooses two plaintext messages

Round 3: Charlie chooses a random binary number  $b \leftarrow_R \{0, 1\}$

Round 4: Charlie encrypts the corresponding message

$c = Enc_k(m_b)$

Round 5: Adv guesses the value of  $b$

$b' \in \{0, 1\}$

Adversary wins if  $b = b'$

# Computational secure IND-EAV

- *Enc* is computationally secure if for any Adv running in polynomial time:

$$P(\text{Adv wins}) = \frac{1}{2} + \text{negligible}(|k|)$$

How to achieve this?

Charlie (Challenger)



$k, E_k$

$b \leftarrow_R \{0, 1\}$

Adv



$m_0, m_1 \in \mathcal{M}$

$c = \text{Enc}_k(m_b)$

$b' \in \{0, 1\}$

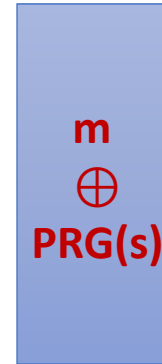
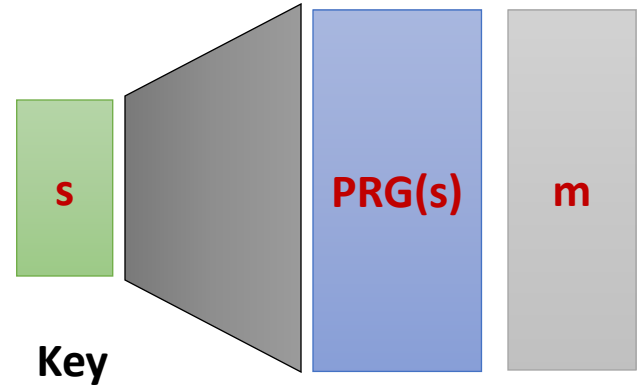
Adversary wins if  $b = b'$

# Using a PRG to build efficient OTP

Use PRGs to “shorten” the key in the one time pad

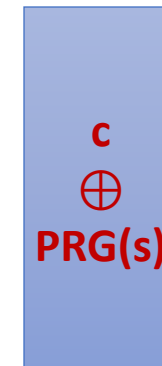
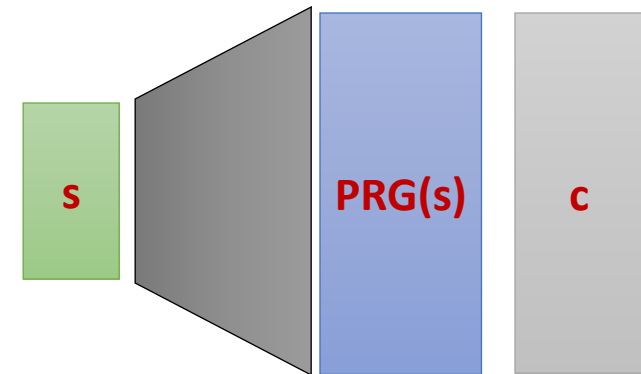
**Key:** random string of length  $n$   
**Plaintexts:** strings of length  $\ell(n)$

**Enc(s,m)**



**STREAM  
CIPHER**

**Dec(s,m)**



Examples:  
RC4, Salsa20

**IND-EAV secure one-time pad**



# Adversarial capability

- **Ciphertext-only attack: Perfect security, IND-EAV**

- Adversary observes one ciphertext
- Cannot infer information about plaintext

- **Chosen-plaintext attack: IND-CPA**

- Adversary can encrypt messages of his choice
- Cannot infer information about plaintext by observing ciphertext

- **Chosen-ciphertext attack: IND-CCA**

- Adversary can decrypt ciphertexts of its choice
- Cannot learn plaintext information on other ciphertext



Stronger  
attacker

# IND-CPA security

- Ciphertext Indistinguishability under Chosen-Plaintext Attack (CPA)
- Adv can encrypt messages of its choice

Round 1: Charlie chooses  $k$  and encryption algo

Round 2: Adv can encrypt messages

Round 3: Adv chooses two plaintext messages

Round 4: Charlie chooses a random binary number  $b \leftarrow_R \{0, 1\}$

Round 5: Charlie encrypts the corresponding message

Round 6: Adv can encrypt messages

Round 7: Adv guesses the value of  $b$

Adversary wins if  $b = b'$

Charlie



$k, Enc_k$

Query: Encrypt  $m$



Reply: Ciphertext  $c$



Adv



$m_0, m_1 \in \mathcal{M}$



$c = Enc_k(m_b)$



Query: Encrypt  $m$



Reply: Ciphertext  $c$



$b' \in \{0, 1\}$

# IND-CPA Security

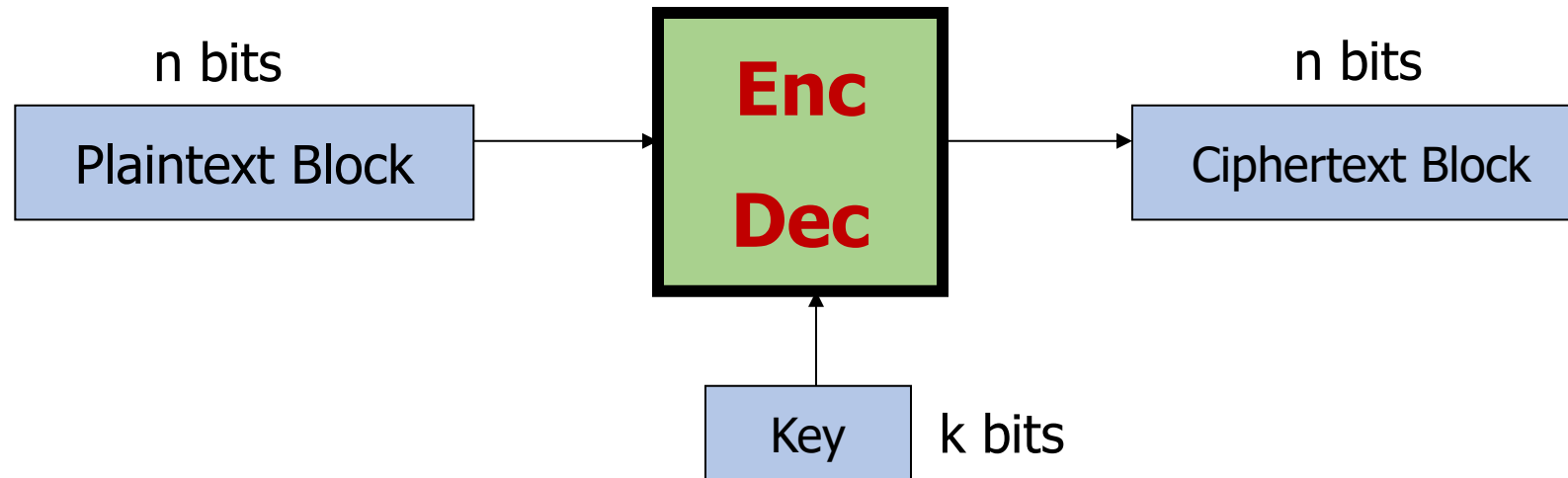
- Adversary can encrypt messages of his choice
  - Including  $m_0, m_1$
- Adversary can encrypt any message before and after seeing the ciphertext  $c$
- CPA adversary is stronger than EAV
- A scheme secure under CPA is also secure under EAV
- But not the other way around!
  - The One-time pad is IND-EAV secure, but not IND-CPA secure
  - IND-CPA is strictly stronger than IND-EAV (for symmetric-key encryption)
- How to design IND-CPA secure ciphers?

# Symmetric Block Ciphers

# Symmetric Key Cryptography

- Algorithms that use a single key for encryption and decryption
  - i.e. the algorithm is reversible
  - $\forall k \forall m \text{ } Dec_k(Enc_k(m)) = m$  where  $m$  is a message,  $k$  is a key, and  $Dec_k$  and  $Enc_k$  are decryption and encryption using  $k$
- Historic examples:
  - Caesar shift, mono and polyalphabetic substitution, OTP
- Modern examples (block ciphers):
  - DES, 3DES, RC4, Blowfish, Twofish, AES
  - **Warning**: many of these methods are known to be vulnerable

# Block ciphers: crypto work horse



Canonical examples:

1. **DES**:  $n=64$  bits,  $k=56$  bits
2. **Triple DES**:  $n=64$  bits,  $k=168$  bits
3. **AES**:  $n=128$  bits,  $k=128, 192, 256$  bits

Desired properties:

1. Change one bit of plaintext completely changes ciphertext
2. Good mixing properties
3. Ciphertext looks random

# The Data Encryption Standard (DES)

- **Early 1970s**: Horst Feistel designs Lucifer at IBM  
key-len = 128 bits ; block-len = 128 bits
- **1973**: NBS asks for block cipher proposals.  
IBM submits variant of Lucifer.
- **1976**: NBS adopts DES as a federal standard  
key-len = 56 bits ; block-len = 64 bits
- **1997**: DES broken by exhaustive search (short keys)
- **2000**: NIST adopts Rijndael as AES to replace DES

# Data Encryption Standard (DES)

- Designed by IBM, with modifications proposed by the NSA
- US national standard from 1977 to 2001
- Block size is 64 bits
- Key size is 56 bits
- Has 16 rounds based on Feistel permutations
- Designed mostly for fast implementation in hardware
  - Software implementation is somewhat slow
- Considered insecure now
  - Vulnerable to brute-force attacks, key too short



# Advanced Encryption Standard (AES)

- In 1997, NIST made a formal call for algorithms stipulating that the AES would specify an unclassified, publicly disclosed encryption algorithm, available royalty-free, worldwide
- Goal: replace DES for both government and private-sector encryption.
- The algorithm must implement symmetric key cryptography as a block cipher and (at a minimum) support block sizes of 128-bits and key sizes of 128-, 192-, and 256-bits.
- In 1998, NIST selected 15 AES candidate algorithms.
- In 2000, NIST selected Rijndael (invented by Joan Daemen and Vincent Rijmen) as the AES
- Designed to be efficient in both hardware and software

# AES Example



Alice



Eavesdropper



Bob



$M$



$Enc_{K_{AES}}(M)$



$M$



$Enc_{K_{AES}}(M)$

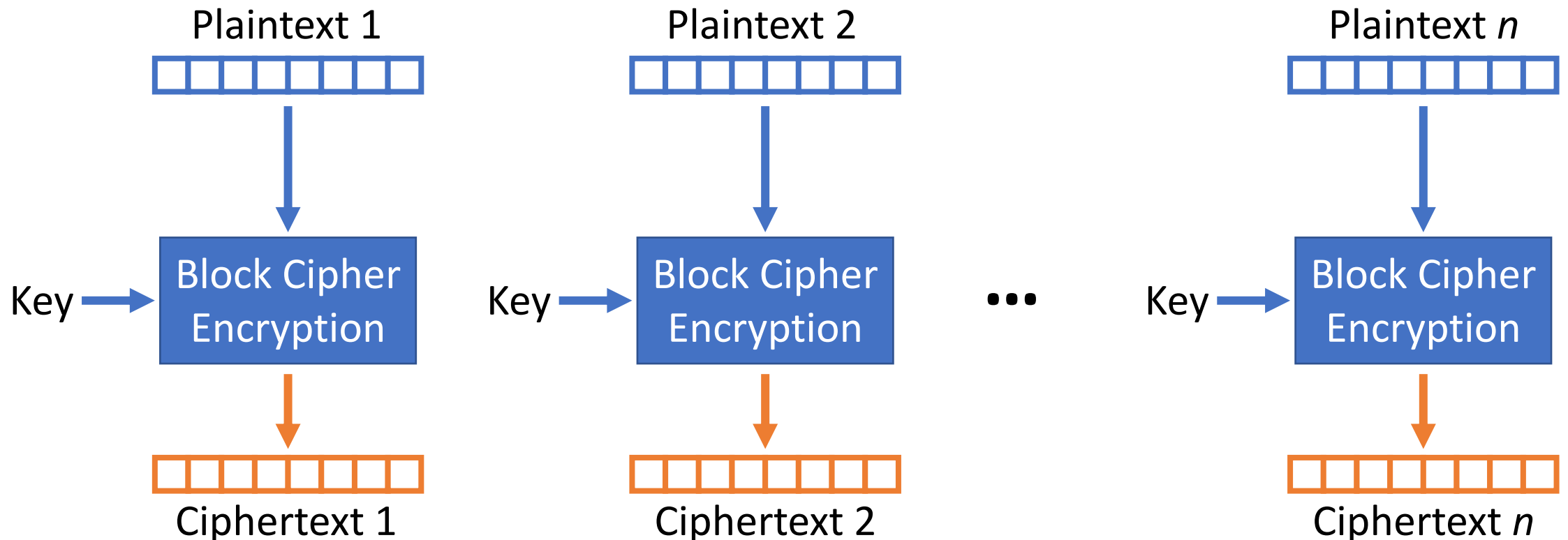
- AES is assumed to be secure (aka ciphertext is pseudorandom)!
- This is backed up by years of cryptanalysis
- Block cipher: encrypts blocks of fixed size

# Need for Encryption Modes

- A block cipher encrypts only one block
  - But a message may be longer than one block
- Need a way to extend the algorithm to encrypt arbitrarily long messages
- Need to ensure that if block cipher is secure, then whole encryption is secure
  - Whole operation should be secure if block cipher is secure

# ECB Encryption Mode

- Message is broken into independent blocks
- Electronic Code Book (ECB): each block is encrypted separately



# Cryptanalysis of ECB Mode

- Deterministic
  - The same data block always gets encrypted the same way
    - Reveals patterns when data repeats!
  - $m$  encrypted with  $k$  always produces the same  $c$
  - This is the same problem we had with the Vigenère cipher
- Is the ECB mode IND-CPA secure?
- Is the ECB mode IND-EAV secure?
- **Do not use ECB mode in practice**



# Lessons on IND-CPA Security

- ECB uses deterministic encryption
  - Encryption of a message  $m$  is always the same
  - Adv can trivially win the IND-CPA game
- **Deterministic encryption is not IND-CPA secure!**
- CPA secure encryption needs to be randomized!
  - How is that achieved?