CY 7790

Special Topics in Security and Privacy: Machine Learning Security and Privacy
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Deep Learning with Differential Privacy

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Motivation for Differential Privacy

- **Anonymization is not enough!**
    - Netflix Prize
        - Contestants were given a dataset of user ratings where identifying information was anonymized and competed to create a recommendation engine
        - Researchers were able to de-anonymize the dataset by linking user ratings to public IMDB ratings
- Ideally, we need a way to learn general trends of the dataset without revealing any individual's private information/contribution

Intuition Behind Differential Privacy

- Suppose we have an algorithm/model, $\mathscr{M}$, which outputs the probability of a student, x, having an F in CY7790 ($\mathscr{M}(x) \in [0,1]$)

  - $\mathscr{M}$ is trained on dataset D, and when we make the query $\mathscr{M}_D(\text{John})$, the output is 0.55

  - $\mathscr{M}$ is trained on dataset D + John, and when we make the query $\mathscr{M}_{D+John}(\text{John})$, the output is 0.57. So it is unclear whether John is failing CY7790

[Mukul Rathi]

Intuition Behind Differential Privacy

- What if $\mathscr{M}_{D+John}$(John) outputs 0.80?
  - Then we have high confidence that John is failing CY7790
  - We get higher accuracy on our test set at the cost of privacy:

Privacy loss

$$log(\frac{\mathbb{P}[\mathscr{M}_{D+John}(John) = 1]}{\mathbb{P}[\mathscr{M}_D(John) = 1]}) \quad \Rightarrow \quad log(\frac{0.57}{0.55}) \approx 0.036 \quad\quad log(\frac{0.80}{0.55}) \approx 0.375$$

[Mukul Rathi]

How do we bound this privacy loss?

Differential Privacy

- Let D, D' be neighboring datasets ($\| D - D' \|_1 \leq 1$), E be some potential output of $\mathscr{M}$

- Then, using the definition of privacy loss from before:

$$log(\frac{\mathbb{P}[\mathscr{M}(D) \in E]}{\mathbb{P}[\mathscr{M}(D') \in E]}) \leq \epsilon$$

Privacy budget

(i.e. the maximum possible privacy loss)

$$\Rightarrow \frac{\mathbb{P}[\mathscr{M}(D) \in E]}{\mathbb{P}[\mathscr{M}(D') \in E]} \leq e^{\epsilon}$$

Differential Privacy

- Let D, D' be neighboring datasets ($\| D - D' \|_1 \leq 1$), E be some potential output of $\mathscr{M}$

- Then, using the definition of privacy loss from before:

<u>ϵ-Differential Privacy</u>

$$\mathbb{P}[\mathscr{M}(D) \in E] \leq e^{\epsilon} \cdot \mathbb{P}[\mathscr{M}(D') \in E]$$

Differential Privacy

$(\epsilon - \delta)$-Differential Privacy

$$\mathbb{P}[\mathscr{M}(D) \in E] \leq e^{\epsilon} \cdot \mathbb{P}[\mathscr{M}(D') \in E] + \boxed{\delta}$$

Failure probability

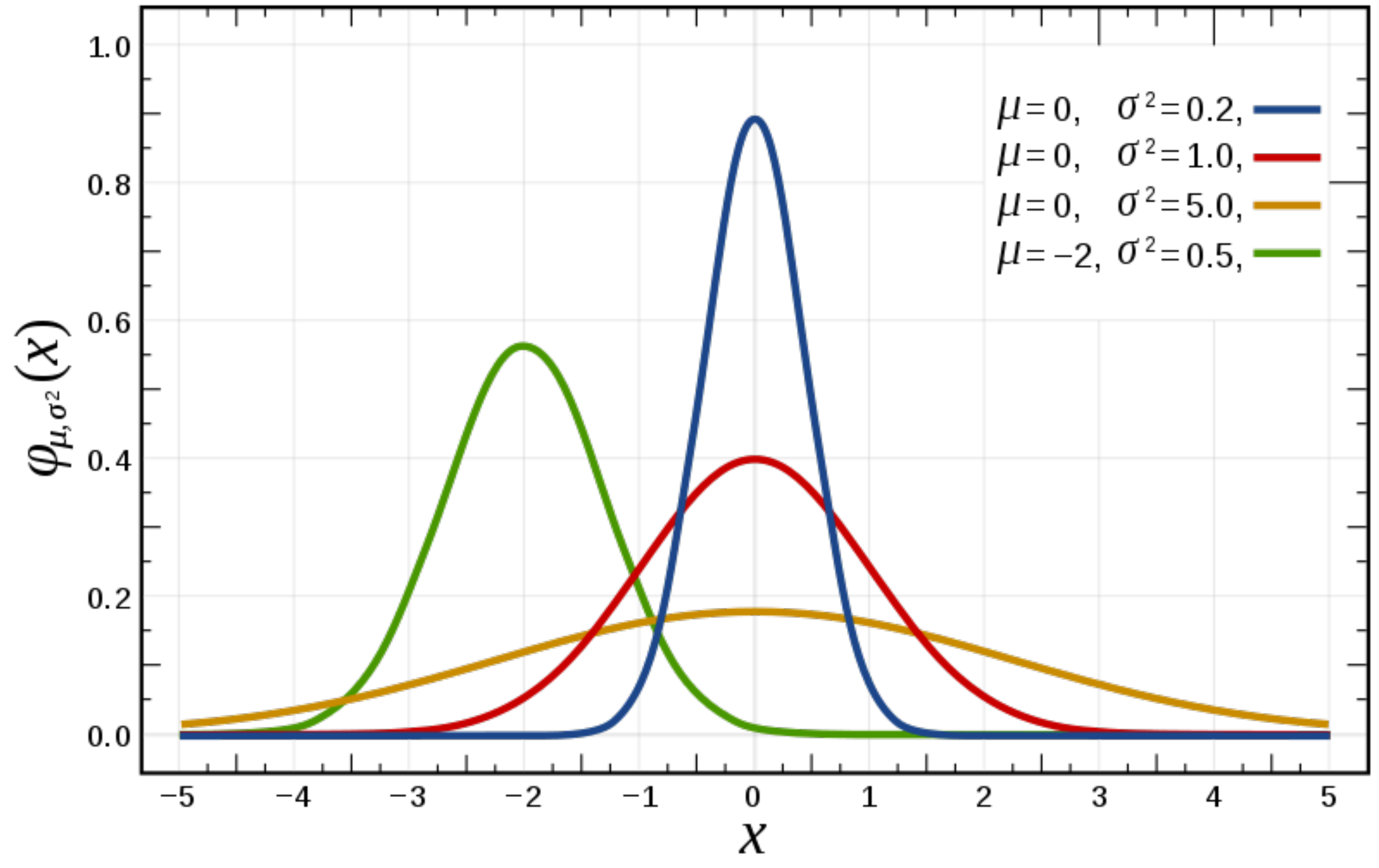
(i.e. our algorithm is ϵ-DP with probability $1 - \delta$)

Differential Privacy in Practice

- How do we achieve differential privacy?
  - Just add noise… intelligently!
  - **Sensitivity:** Given a function $f : D \to \mathbb{R}$, we can define the S_f as $|f(x) - f(x')|$ where x, x' are neighboring points
- **The Gaussian Mechanism:**

$$\mathscr{M}(x) \triangleq f(x) + \mathscr{N}(0,\ S_f^2 \cdot \sigma)$$

  - $(\epsilon - \delta)$-DP for $\delta \geq \frac{4}{5}\exp(-(\sigma\epsilon)^2/2), \quad \epsilon < 1$



Properties

- **Composition:** If we run k, $(\epsilon - \delta)$-DP algorithms:
  - The resulting algorithm will be $(k\epsilon - k\delta)$-DP
- **Advanced composition:** If $k < 1/\epsilon^2$ The resulting algorithm is $(O(\sqrt{klog(1/\delta')}) \cdot \epsilon - k\delta + \delta')$-DP for all $\delta' > 0$
- **Amplification:** If we sample a fraction, q, of the data, our algorithm becomes $(q\epsilon - q\delta)$-DP

DP-SGD Algorithm

Goal: Minimize privacy loss at each iteration.
(Minimizes composed privacy loss)

1. Compute gradient of random sample

2. Clip gradient (i.e. force max ℓ_2 norm of gradient to be C in order to "bound sensitivity of the gradient")

3. Add noise calibrated to the clipping norm, C, times the noise scale, σ

4. Update θ

Algorithm 1 Differentially private SGD (Outline)

Input: Examples $\{x_1, \ldots, x_N\}$, loss function $\mathcal{L}(\theta) = \frac{1}{N}\sum_i \mathcal{L}(\theta, x_i)$. Parameters: learning rate η_t, noise scale σ, group size L, gradient norm bound C.

Initialize θ_0 randomly

for $t \in [T]$ **do**

Take a random sample L_t with sampling probability L/N

Compute gradient

For each $i \in L_t$, compute $\mathbf{g}_t(x_i) \leftarrow \nabla_{\theta_t}\mathcal{L}(\theta_t, x_i)$

Clip gradient

$\bar{\mathbf{g}}_t(x_i) \leftarrow \mathbf{g}_t(x_i)/\max\left(1, \frac{\|\mathbf{g}_t(x_i)\|_2}{C}\right)$

Add noise

$\tilde{\mathbf{g}}_t \leftarrow \frac{1}{L}\left(\sum_i \bar{\mathbf{g}}_t(x_i) + \mathcal{N}(0, \sigma^2 C^2 \mathbf{I})\right)$

Descent

$\theta_{t+1} \leftarrow \theta_t - \eta_t \tilde{\mathbf{g}}_t$

Output θ_T and compute the overall privacy cost (ε, δ) using a privacy accounting method.

Analysis

- **Naive Approach**
  - We can use basic composition to arrive at an upper bound for our privacy budget
    - $(T\epsilon - T\delta)$-DP
  - Additionally, we can apply amplification to get a tighter bound on this budget
    - $(Tq\epsilon - Tq\delta)$-DP

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Descent

$\theta_{t+1} \leftarrow \theta_t - \eta_t \tilde{\mathbf{g}}_t$

Output θ_T and compute the overall privacy cost (ε, δ) using a privacy accounting method.

Analysis

- **Using Advanced Composition**
  - $(O(q\epsilon \cdot \sqrt{Tlog(1/\delta)}) - Tq\delta)$-DP
  - So, our privacy budget is a function of $\sqrt{Tlog(1/\delta)}$ instead of T

Algorithm 1 Differentially private SGD (Outline)

Input: Examples $\{x_1, \ldots, x_N\}$, loss function $\mathcal{L}(\theta) = \frac{1}{N}\sum_i \mathcal{L}(\theta, x_i)$. Parameters: learning rate η_t, noise scale σ, group size L, gradient norm bound C.

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Descent

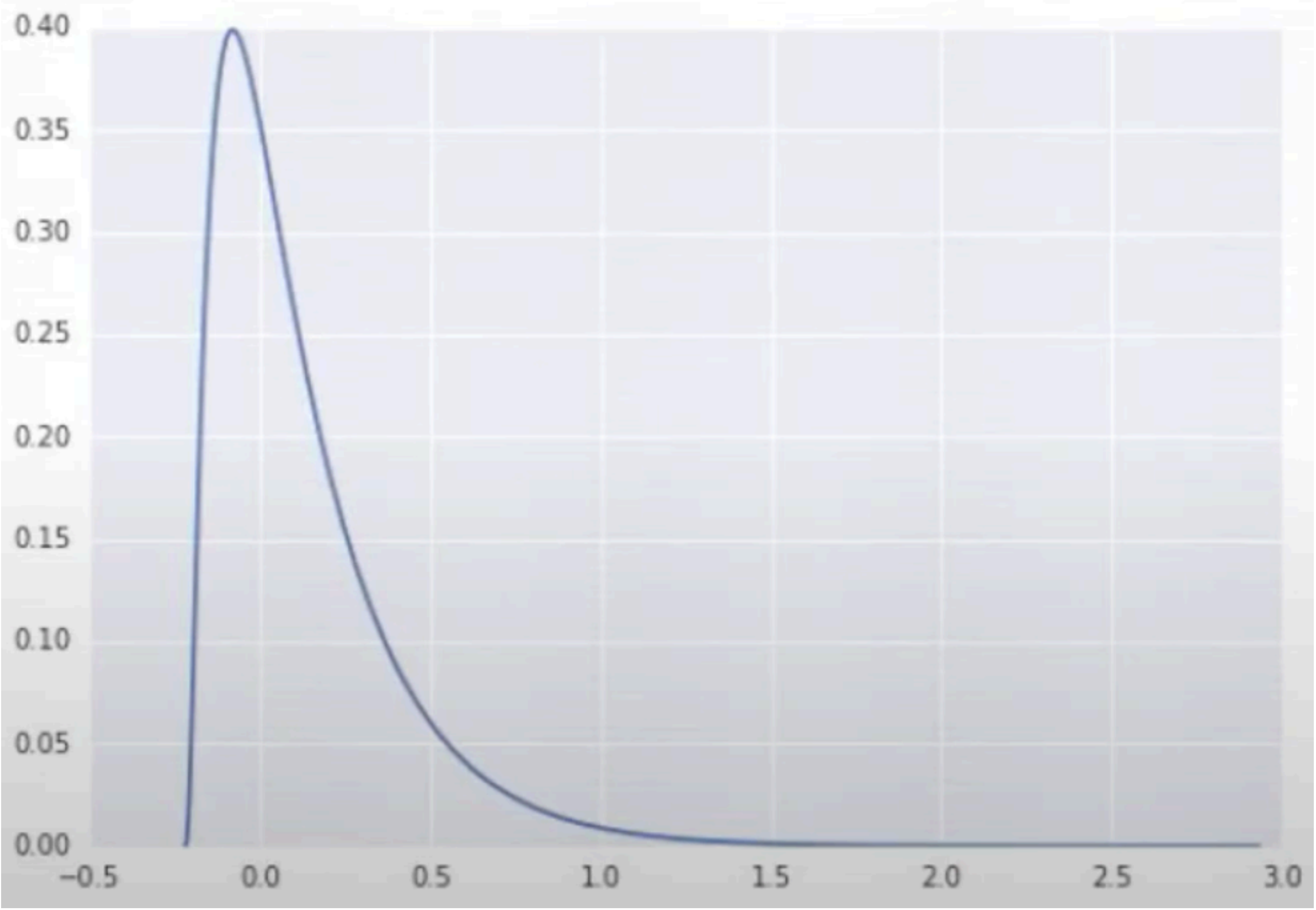
$\theta_{t+1} \leftarrow \theta_t - \eta_t \tilde{\mathbf{g}}_t$

Output θ_T and compute the overall privacy cost (ε, δ) using a privacy accounting method.

Does advanced composition give us the best bound?

Analysis

- **The Moments Accountant**
  - The authors decide to treat privacy loss as its own random variable
  - The privacy loss has a quick drop-off but has a very long tail
    - So, we can use **concentration inequalities** to bound privacy loss



log(privacy loss)

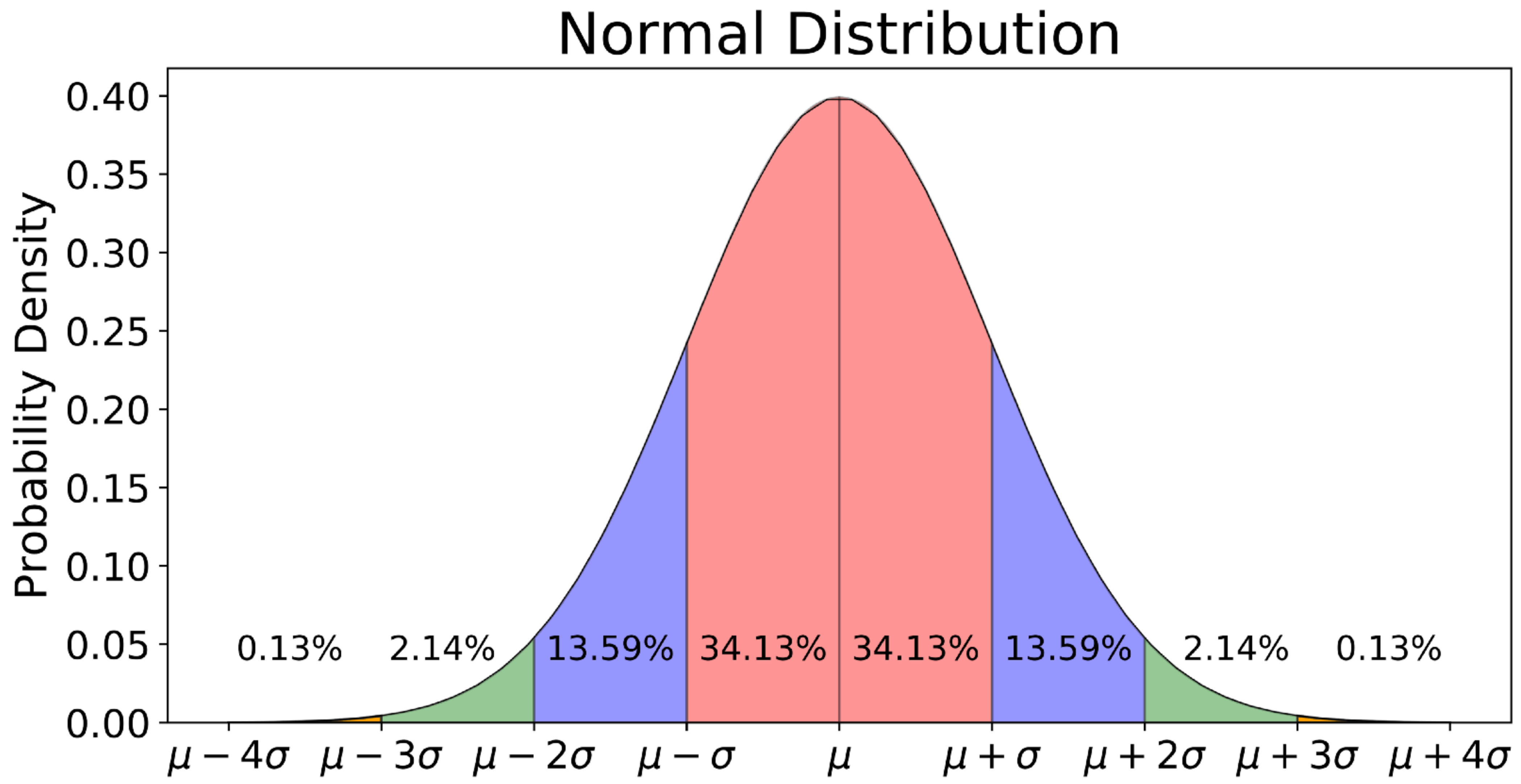
Analysis

- **Concentration Inequalities**
  - Markov's Inequality:

$$\mathbb{P}[X \geq a] \leq \frac{\mathbb{E}[X]}{a}$$

  - Chebyshev's Inequality:

$$\mathbb{P}[\,|X-\mu| \geq a] \leq \frac{\mathbb{E}[\,|X-\mu|^2]}{a^2} = \frac{var(X)}{a^2}$$



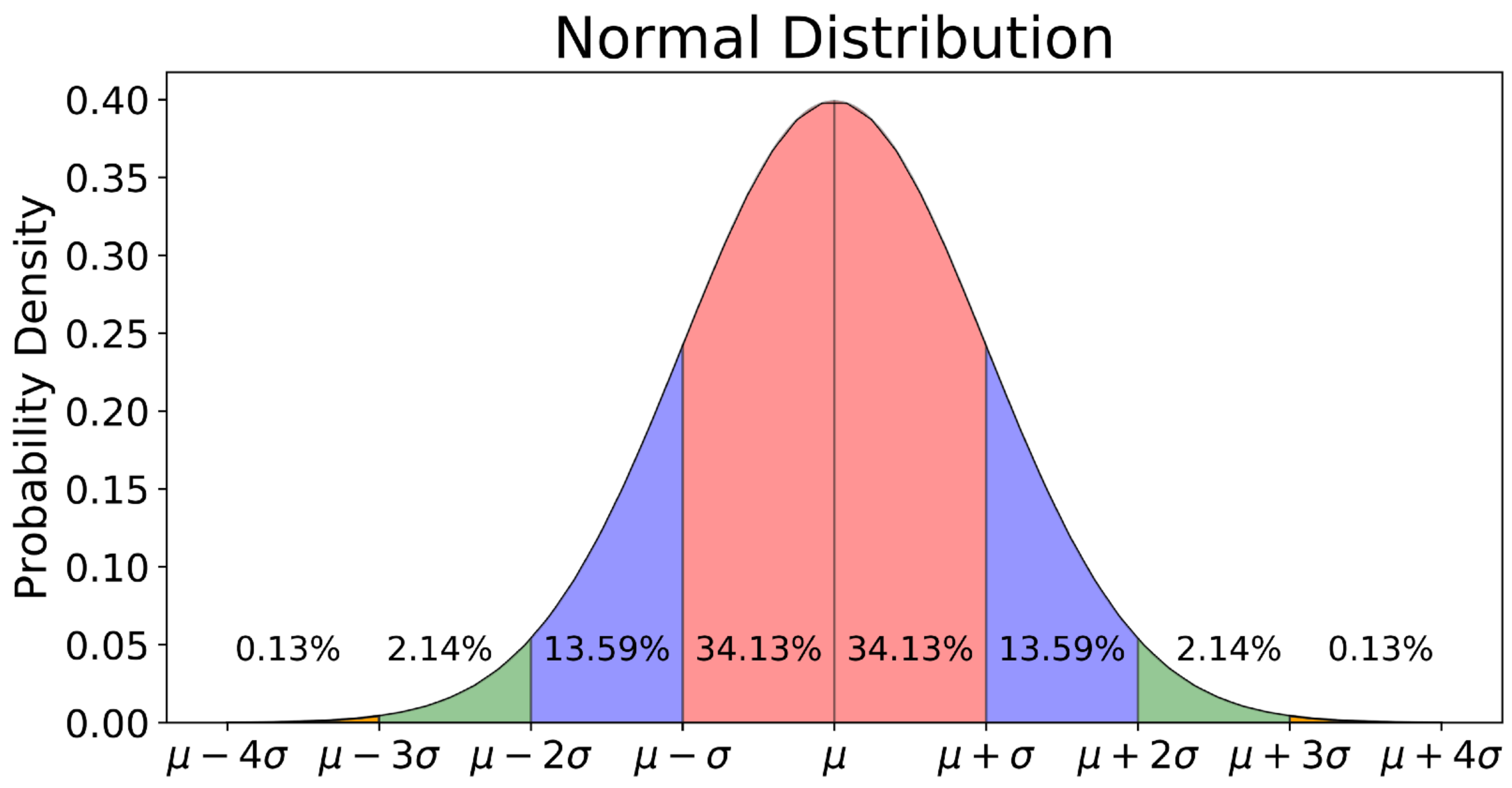
Analysis

- **Concentration Inequalities**
  - We can extend these inequalities to higher order moments using the moment generating function:

$$\mathbb{P}[\,|X-\mu|\geq a] \leq \frac{\mathbb{E}[\,|X-\mu|^k]}{a^k}$$

$$\downarrow$$

$$\mathbb{P}[(X-\mu)\geq a] = \mathbb{P}[e^{\lambda(X-\mu)} \geq a] \leq \frac{\mathbb{E}[e^{\lambda(X-\mu)}]}{e^{\lambda a}}$$



Analysis

- **Concentration Inequalities**
  - Then, we can optimize our choice of λ to obtain the tightest result
  - (For practical reasons, the authors only keep track of the first 32 moments)
- Using this approach, the authors find that the algorithm is $(q\epsilon\sqrt{T} - \delta)$-DP

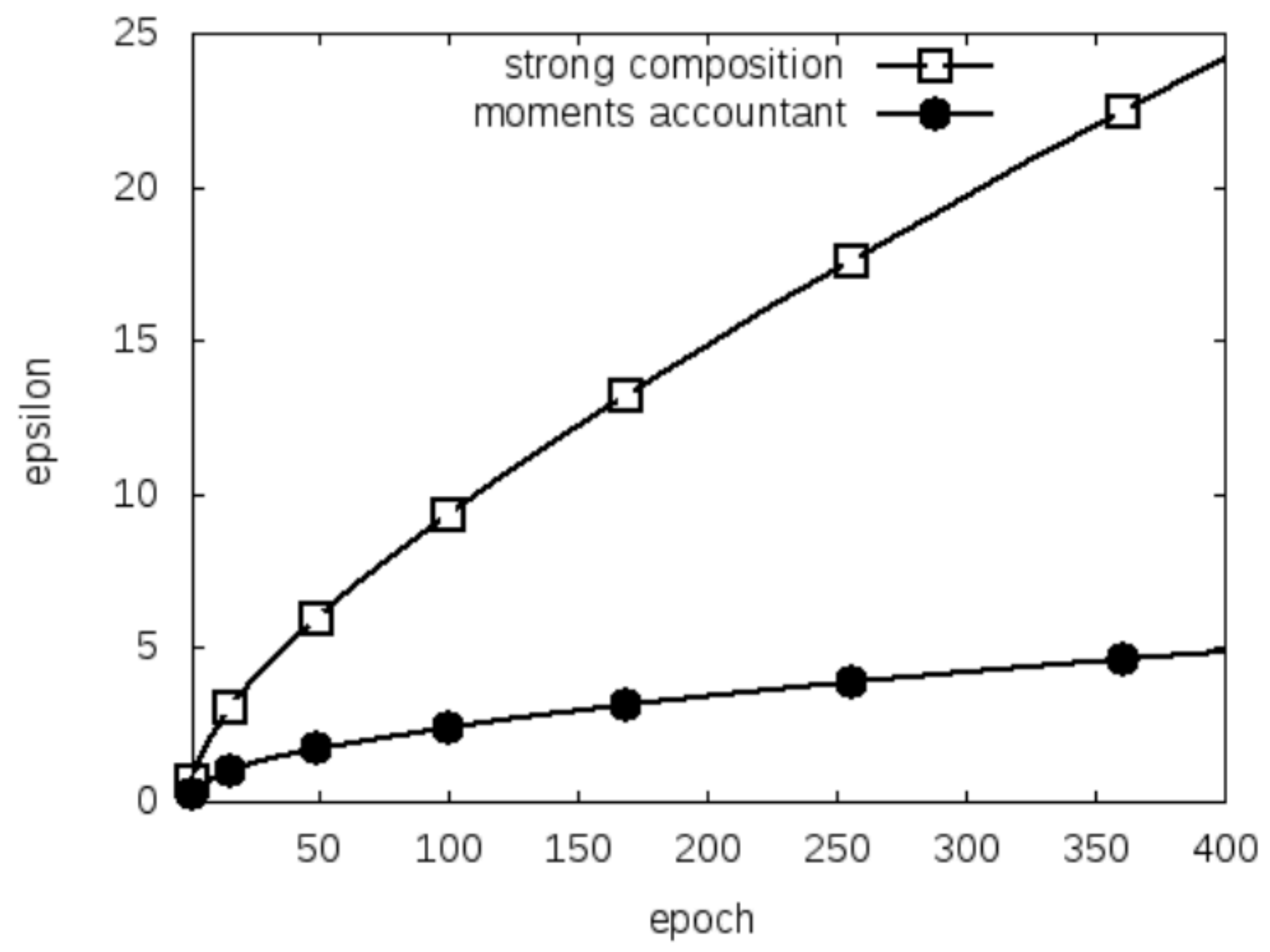


Figure 2: The ε value as a function of epoch E for $q = 0.01$, $\sigma = 4$, $\delta = 10^{-5}$, using the strong composition theorem and the moments accountant respectively.

Empirical Results

- Using a simple feed-forward neural network trained on MNIST the authors achieved the following results
  - No privacy $\rightarrow$ 98.3% accuracy
  - $\epsilon = 8,\ \delta = 10^{-5}$ $\rightarrow$ 97% accuracy (Compared to 80% in previous work)
  - $\epsilon = 2,\ \delta = 10^{-5}$ $\rightarrow$ 95% accuracy
  - $\epsilon = 0.5,\ \delta = 10^{-5}$ $\rightarrow$ 90% accuracy

Empirical Results

- Using a simple feed-forward neural network trained on CIFAR10 the authors achieved the following results
  - No privacy → 80% accuracy
  - $\epsilon = 8,\ \delta = 10^{-5}$ → 73% accuracy
  - $\epsilon = 2,\ \delta = 10^{-5}$ → 67% accuracy

Strengths

- The authors show that differentially private deep learning can achieve good accuracy with a reasonable amount of privacy leakage
- Moments accountant is a strong tool for computing privacy of iterative algorithms
  - It gives us more room for accuracy without the privacy loss found using advanced composition

Limitations

- A reasonable tradeoff between privacy and accuracy does not seem to hold for more complicated datasets, like CIFAR10
- The authors only consider a simple neural network in their experiments

Auditing Differentially Private ML: How Private is Private SGD

Matthew Jagielski, Jonathan Ullman, Alina Oprea

Harsh Chaudhari

Main Idea

- Investigate the extent to which DP-SGD does or doesn’t give better privacy in practice than what its theoretical counterparts suggest.

- How to Investigate? -> Use data poisoning attacks

Roadmap

- Background:
  - Differential Privacy
  - DP-SGD
  - Backdoor Attack
- Statistically Measuring Differential Privacy
- Poisoning Attacks
- Experiments

Differential Privacy

Definition 2.1 ([DMNS06]). An algorithm $\mathcal{A} : \mathcal{D} \mapsto \mathcal{R}$ is (ε, δ)-*differentially private* if for any two datasets D_0, D_1 which differ on at most one row, and every set of outputs $\mathcal{O} \subseteq \mathcal{R}$:

$$\Pr[\mathcal{A}(D_0) \in \mathcal{O}] \leq e^{\varepsilon} \Pr[\mathcal{A}(D_1) \in \mathcal{O}] + \delta, \tag{1}$$

where the probabilities are taken only over the randomness of $\mathcal{A}$.

Lemma 1 (Group Privacy). Let D_0, D_1 be two datasets differing on at most k rows, $\mathcal{A}$ is an (ε, δ)-differentially private algorithm, and $\mathcal{O}$ an arbitrary output set. Then

$$\Pr[\mathcal{A}(D_0) \in \mathcal{O}] \leq e^{k\varepsilon} \Pr[\mathcal{A}(D_1) \in \mathcal{O}] + \tfrac{e^{k\varepsilon}-1}{e^{\varepsilon}-1} \cdot \delta. \tag{2}$$

Group privacy will give guarantees for poisoning attacks that introduce multiple points.

DP-SGD

Algorithm 1: DP-SGD

Data: Input: Clipping norm C, noise magnitude σ, iteration count T, batch size b, dataset D, initial model parameters θ_0, learning rate η

For $i \in [T]$

    $G = 0$

    For $(x, y) \in$ *batch of* b *random elements of* D

        $g = \nabla_\theta \ell(\theta_i; (x, y))$

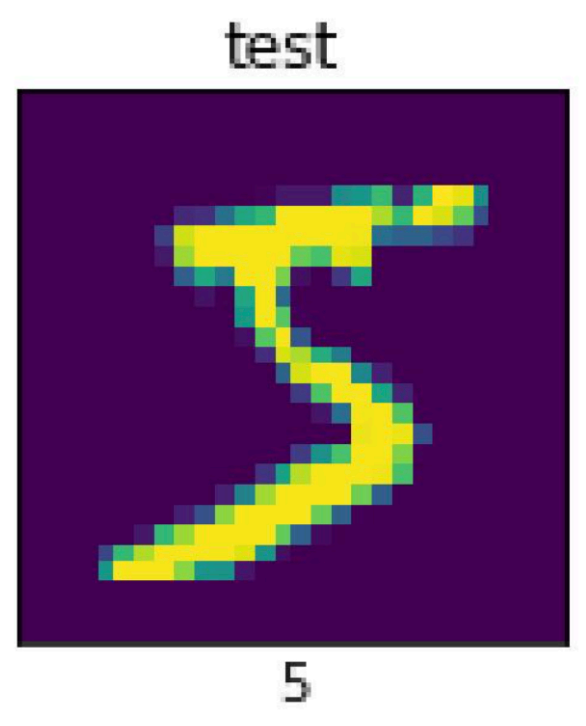
        $G = G + b^{-1} g \cdot \min(1, C||g||_2^{-1})$ ← Gradient Clipping

    $\theta_i = \theta_{i-1} - \eta(G + \mathcal{N}(0, (C\sigma)^2 \mathbb{I}))$ ← Adding Noise

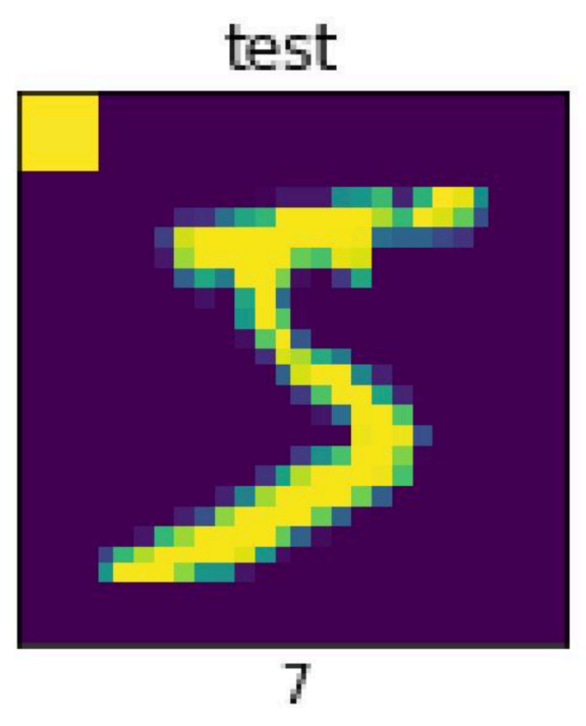
Return θ_T

Backdoor Attack

- Introduce a Backdoor pattern on images during training phase with the desired target label.



- Apply the backdoor pattern on test data and check if the test samples get misclassified to the target label.



Statistically Measuring Differential Privacy

- Given algorithm A, adjacent datasets D_0 and D_1 and outputs O:

$$\Pr[\mathcal{A}(D_0) \in \mathcal{O}] \leq e^{\varepsilon} \Pr[\mathcal{A}(D_1) \in \mathcal{O}]$$

- Begin by computing probabilities:

$$p_0 = \Pr[\mathcal{A}(D_0) \in \mathcal{O}] \qquad p_1 = \Pr[\mathcal{A}(D_1) \in \mathcal{O}]$$

- Lower bound on Epsilon:

$$\varepsilon_{LB} = \ln(p_0/p_1)$$

Statistically Measuring Differential Privacy

- To approximate p_0 and p_1, use Monte Carlo Estimation
- Clopper Pearson confidence intervals: Produce epsilon lower bound with probability 99%

Algorithm 2: Empirically Measuring ε

Data: Algorithm $\mathcal{A}$, datasets D_0, D_1 at distance k, output set $\mathcal{O}$, trial count T, confidence level α

$\mathrm{ct}_0 = 0, \mathrm{ct}_1 = 0$

For $i \in [T]$

  If $\mathcal{A}(D_0) \in \mathcal{O}$ $\mathrm{ct}_0 = \mathrm{ct}_0 + 1$

  If $\mathcal{A}(D_1) \in \mathcal{O}$ $\mathrm{ct}_1 = \mathrm{ct}_1 + 1$

$\hat{p}_0 = \textsc{ClopperPearsonLower}(\mathrm{ct}_0, T, \alpha/2)$

$\hat{p}_1 = \textsc{ClopperPearsonUpper}(\mathrm{ct}_1, T, \alpha/2)$

Return $\varepsilon_{LB} = \ln(\hat{p}_0/\hat{p}_1)/k$

Statistically Measuring Differential Privacy

Theorem 2. When provided with black box access to an algorithm $\mathcal{A}$, two datasets D_0 and D_1 differing on at most k rows, an output set $\mathcal{O}$, a trial number T and statistical confidence α, if Algorithm 2 returns ε_{LB}, then, with probability $1-\alpha$, $\mathcal{A}$ does not satisfy ε'-DP for any $\varepsilon' < \varepsilon_{LB}$.

Proof of Theorem 2. First, the guarantee of the Clopper-Pearson confidence intervals is that, with probability at least $1-\alpha$, $\hat{p}_0 \leq p_0$ and $\hat{p}_1 \geq p_1$, which implies $p_0/p_1 \geq \hat{p}_0/\hat{p}_1$. Second, if $\mathcal{A}$ is ε-DP, then by group privacy we would have $p_0/p_1 \leq \exp(k\varepsilon)$, meaning $\mathcal{A}$ is *not* ε'-DP for any $\varepsilon' < \frac{1}{k}\ln(p_0/p_1)$. Combining the two statements, $\mathcal{A}$ is *not* ε' for any $\varepsilon' < \frac{1}{k}\ln(\hat{p}_0/\hat{p}_1) = \varepsilon_{LB}$. $\square$

Poisoning Attack: Backdoor

Algorithm 3: Baseline Backdoor Poisoning Attack and Test Statistic (Section 3.1)

Data: Dataset X, Y, poison size k, perturbation function $Pert$, target class y_p

Function $\textsc{Backdoor}(X, Y, k, Pert, y_p)$**:**

$X_p = \textsc{GetRandomRows}(X, k)$

$X_p' = Pert(X_p)$

$X_{tr}^p = \textsc{ReplaceRandomRows}(X, X_p')$

$Y_{tr}^p = \textsc{ReplaceRandomRows}(Y, y_p)$

return $D_0 = (X, Y), D_1 = (X_{tr}^p, Y_{tr}^p)$

Data: Model f, dataset (X, Y), pert. function $Pert$, target class y_p, loss function ℓ, threshold Z

Function $\textsc{BackdoorTest}(f, X, Y, Pert, y_p, \ell, Z)$**:**

$X_p = Pert(X)$

If $\sum_{x_p \in X_p} \ell(f; (x_p, y_p)) > Z$ **Return** Backdoored

Return Not Backdoored

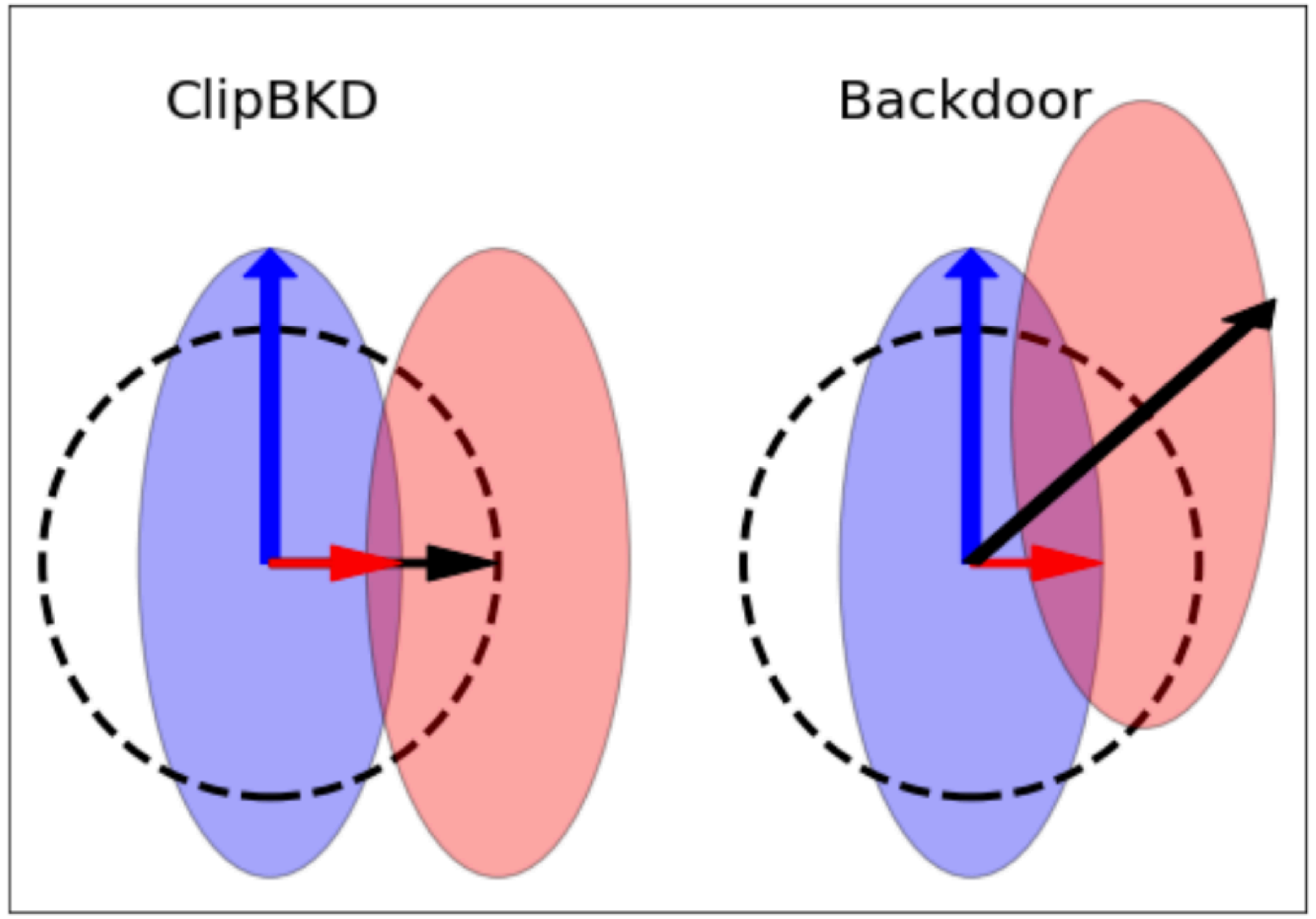
Clipping-Aware Backdoor

- Gradient of model parameters with respect to a poisoning point is

$$\nabla_w \ell(w, b; x_p, y_p) = \ell'(w \cdot x_p + b, y_p) x_p.$$

- For Backdoor attacks:
    - Large $|\ell'(w \cdot x_p + b, y_p)|$
    - Relationship gets broken after clipping
- Clipping-Aware Backdoor:
    - Choose x_p such that it minimizes

$$Var_{(x,y)\in D}\left[\ell'(w \cdot x_p + b, y_p)x_p \cdot \ell'(w \cdot x + b, y)x\right] \leq Var_{(x,y)\in D}[x_p \cdot x].$$



Clipping-Aware Backdoor

Algorithm 4: Clipping-Aware Backdoor Poisoning Attack and Test Statistic (Section 3.2)

Data: Dataset X, Y, pretrained model f, poison size k, dataset dimension d, norm m

Function $\textsc{ClipBkd}(X, Y, k, f, m)$:

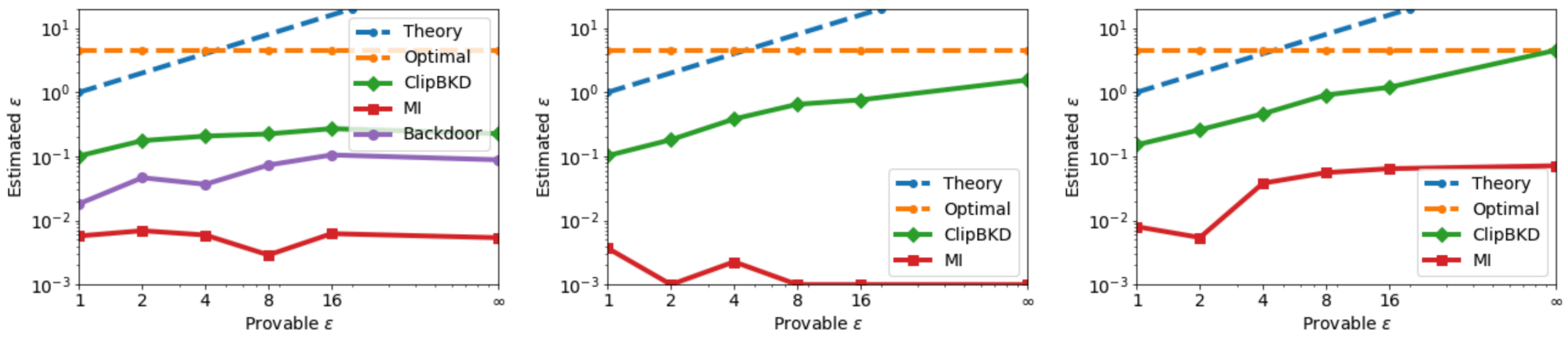
- $U, D, V = SVD(X)$ ▷ Singular value decomposition
- $x_p = mV_d$ ▷ V_d is the singular vector for smallest singular value
- $y_p = \arg\min_i f(x_p)$ ▷ Pick class maximizing gradient norm
- $X_{tr}^p = \textsc{ReplaceRandomRows}(X, [x_p] * k)$ ▷ Add poisoning point k times
- $Y_{tr}^p = \textsc{ReplaceRandomRows}(Y, [y_p] * k)$ ▷ Add targeted class k times
- **return** $D_0 = (X, Y), D_1 = (X_{tr}^p, Y_{tr}^p)$

Data: Model f, Poison Data x_p, y_p, Threshold Z

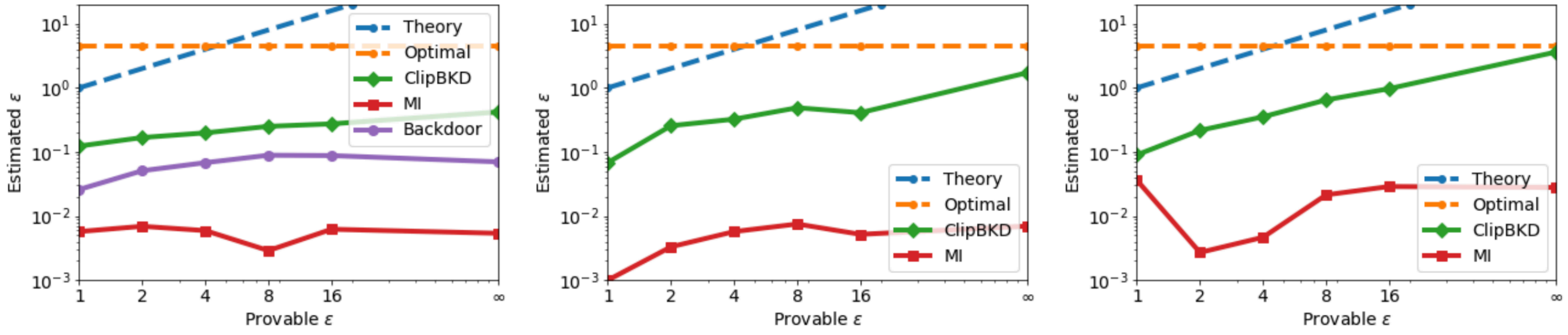
Function $\textsc{ClipBkdTest}(f, x_p, y_p, Z)$:

- **If** $(f(x_p) - f(0^d)) \cdot y_p > Z$ **Return** Backdoored
- **Return** Not Backdoored

Experiments



(a) FMNIST, LR (b) CIFAR10, LR (c) P100, LR



(d) FMNIST, FNN (e) CIFAR10, FNN (f) P100, FNN

Figure 2: Performance of privacy attacks MI, Backdoor, and ClipBKD on our datasets. LR = logistic regression, FNN = two-layer neural network. Backdoor attacks have not been developed for Purchase-100, so only MI and Clip-BKD were run. Backdoors do not provide positive ε_{LB} on CIFAR10 due to difficulty with the pretrained model.

Experiments

Params	Fixed Init	Init Rand $= 0.5\sigma$	Init Rand $= \sigma$	Init Rand $= 2\sigma$
$\varepsilon_{th} = 1, \sigma_{GD} = 5.02$	0.13 / 0.15 / 0.13	0.13 / 0.17 / 0.13	0.06 / 0.12 / 0.09	0.01 / 0.06 / 0.08
$\varepsilon_{th} = 2, \sigma_{GD} = 2.68$	0.33 / 0.37 / 0.28	0.27 / 0.33 / 0.39	0.10 / 0.17 / 0.27	0.01 / 0.06 / 0.17
$\varepsilon_{th} = 4, \sigma_{GD} = 1.55$	0.89 / 0.75 / 0.71	0.28 / 0.52 / 0.78	0.08 / 0.20 / 0.54	0.02 / 0.10 / 0.18
$\varepsilon_{th} = 8, \sigma_{GD} = 1.01$	1.61 / 1.85 / 1.90	0.33 / 0.55 / 1.27	0.07 / 0.25 / 0.53	0.01 / 0.05 / 0.20
$\varepsilon_{th} = 16, \sigma_{GD} = 0.73$	2.15 / 2.16 / 2.43	0.36 / 0.80 / 1.39	0.13 / 0.27 / 0.72	0.02 / 0.08 / 0.16
$\varepsilon_{th} = \infty, \sigma_{GD} = 0$	4.54 / 4.54 / 4.54	0.29 / 0.95 / 2.36	0.10 / 0.42 / 0.79	0.03 / 0.09 / 0.27

Table 2: Lower bound ε_{LB} measured with ClipBKD for clipping norms of (0.5 / 1 / 2) for two-layer neural networks trained on FMNIST. Training accuracy for all models is 96%-98%. Results are the maximum over $k = 1, 2, 4, 8$. σ_{GD} refers to the DP-SGD noise multiplier, while σ is Glorot initialization randomness [GB10]. All reported values of ε_{LB} are valid with 99% confidence over the randomness of our experiments.

Thank You