MULTIPLE LINEAR REGRESSION

P1 $w, x : (d \times 1)$; $w = \begin{bmatrix} w_1 \\ \vdots \\ w_d \end{bmatrix}$ $\quad x = \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}$

$$w^T x = \boxed{w_1 x_1 + \dots + w_d x_d}$$

$$\underbrace{\frac{\partial w^T x}{\partial x}}_{\text{DEF}} = \left[\frac{\partial w^T x}{\partial x_1} \ \cdots\cdots \ \frac{\partial w^T x}{\partial x_d} \right] = \underbrace{[w_1 \ w_2 \cdots w_d]}_{= w^T}$$

P5 $x = \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}$

$$\boxed{\frac{\partial \|x\|^2}{\partial x}} = \left[\frac{\partial \|x\|^2}{\partial x_1} \ \cdots\cdots \ \frac{\partial \|x\|^2}{\partial x_d} \right]$$

$$\|x\|^2 = \underbrace{x_1^2 + \dots + x_d^2}$$

$$= [2x_1 \dots \ 2x_d]$$

$$= 2x^T$$

MSE: $J(\theta) = \frac{1}{N} \|\underbrace{X\theta - y}\|^2 \in \mathbb{R}$ $\quad \begin{cases} X : N \times (d+1) \\ Y : N \times 1 \\ \theta : d \times 1 \end{cases}$

$\|\theta\|^2$

$$\frac{\partial J(\theta)}{\partial \theta} = \frac{1}{N} \cdot 2 (X\theta - y)^T \cdot X$$

CHAIN RULE

$(A^T)^T = A$

$(AB)^T = B^T A^T$

$$\frac{\partial J(\theta)}{\partial \theta} = 0$$

$$\underbrace{(X\theta - y)}^{T} \cdot X = 0 \Rightarrow \theta = \dots ?$$

TRANSPOSE

$$X^T(X\theta - y) = 0$$

$$(X^T X)\theta = X^T y$$

$$\boxed{\theta = \underbrace{(X^T X)^{-1}} (X^T y)}$$

If $(X^T X)^{-1}$ exists

$X^T X$: size $(d+1) \times (d+1)$

CLOSED FORM SOLUTION

MULTIPLE LINEAR REGRESSION

$X : (\underline{N} \times 2)$; $X^T X : 2 \times 2$; $\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix}$