LINEAR REGRESSION

INPUT $x_i, y_i \in \mathbb{R}$, $i = \overline{1,N}$; $h_\theta(x) = \theta_0 + \theta_1 x$

$$\rightarrow \mathcal{J}(\theta) = \frac{1}{N}\sum_{i=1}^{N}\left[\theta_0 + \boxed{\theta_1 x_i} - y_i\right]^2$$

$$\boxed{\frac{\partial x^2}{\partial x} = 2x}$$

$$\min_{\theta_0,\theta_1} \mathcal{J}(\theta_0,\theta_1)$$; If θ_0, θ_1 are argmin $\mathcal{J}(\theta)$

$$\Rightarrow \frac{\partial \mathcal{J}(\theta)}{\partial \theta_0} = 0 \ ; \ \frac{\partial \mathcal{J}(\theta)}{\partial \theta_1} = 0$$

$$(1) \quad \frac{\partial \mathcal{J}(\theta)}{\partial \theta_0} = \frac{1}{N}\underbrace{\sum_{i=1}^{N} 2\left[\theta_0 + \theta_1 x_i - y_i\right]}_{} = 0$$

$$(2) \quad \frac{\partial \mathcal{J}(\theta)}{\partial \theta_1} = \frac{1}{N}\sum_{i=1}^{N} 2\left[\theta_0 + \theta_1 x_i - y_i\right]\cdot x_i = 0$$

Notation: $\boxed{\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \ ; \ \bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i}$

$$(1): \quad \theta_0 + \theta_1 \cdot \frac{\sum x_i}{N} - \frac{\sum y_i}{N} = 0 \Rightarrow \underbrace{\theta_0 + \theta_1 \bar{x} = \bar{y}}_{}$$

$$\boxed{\underbrace{\theta_0 = \bar{y} - \theta_1 \bar{x}}_{}}$$

$$(2) \quad \theta_0 \cdot \frac{\sum x_i}{N} + \theta_1 \frac{\sum x_i^2}{N} - \frac{\sum x_i y_i}{N} = 0$$

$$\rightarrow \theta_0 \bar{x} + \theta_1 \cdot \frac{\sum x_i^2}{N} = \frac{\sum x_i y_i}{N}$$

$$N\,(\underbrace{\bar{y} - \theta_1 \bar{x}}_{})\,\bar{x} + \underbrace{\theta_1 \cdot \sum x_i^2}_{} = \sum x_i y_i$$

$$\theta_1\left[\boxed{\sum x_i^2 - N\bar{x}^2}\right] = \boxed{\sum x_i y_i - N\bar{x}\bar{y}}$$

$$\underbrace{\qquad\qquad}_{\shortparallel} \qquad \underbrace{\qquad\qquad}_{\shortparallel}$$

$$\sum (x_i - \bar{x})^2 \qquad \sum (x_i - \bar{x})(y_i - \bar{y})$$

$$\sum_{i=1}^{N} (x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - 2\bar{x} \sum_{i=1}^{N} x_i + N\bar{x}^2$$

$$= \sum_{i=1}^{N} x_i^2 - 2N\bar{x}^2 + N\bar{x}^2$$

$$= \sum_{i=1}^{N} x_i^2 - N\bar{x}^2$$

$$\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{N} x_i y_i - \bar{y} \underbrace{\sum_{i=1}^{N} x_i}_{N\bar{x}} - \bar{x} \underbrace{\sum_{i=1}^{N} y_i}_{N\bar{y}} + N\bar{x}\bar{y}$$

$$= \sum x_i y_i - N\bar{x}\bar{y} - N\bar{x}\bar{y} + N\bar{x}\bar{y}$$

$$= \sum_{i=1}^{N} x_i y_i - N\bar{x}\bar{y}$$

$$\boxed{\theta_1 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})}{\boxed{\sum_{i=1}^{N} (x_i - \bar{x})^2}}}$$

$\longrightarrow \mathrm{Cov}(X, Y)$

$\longrightarrow \mathrm{Var}(X)$

$$\boxed{\theta_0 = \bar{y} - \theta_1 \bar{x}}$$

$\boxed{\text{CHAIN RULE}}$

$$\frac{\partial f(g(x))}{\partial x} = f'(g(x)) \cdot \underbrace{g'(x)}$$

$$\frac{\partial [\theta_0 + \theta_1 x - y]^2}{\partial \theta_0} = 2 \cdot [\theta_0 + \theta_1 x - y]$$

$f(x) = x^2$

$g(\theta_0) = \theta_0 + \theta_1 x - y \quad ; \quad g'(\theta_0) = 1$